# Profiting from Partial Allowance of Ticket Resale

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Whether ticket resale increases or reduces a monopolistic seller's profit is of interest to both business practitioners and researchers. Prior research implicitly deems a resale market a unity, and so argues either for or against it. This paper argues that there is a third option – acknowledging that a resale market is often continuous and buyers often arrive at different times, it is possible to divide the resale market into two segments, then allowing resale in the first segment and denying it in the second. This partial allowance of resale effectively changes time-based price discrimination to type-based price discrimination and may lead to a higher seller profit than under either complete allowance or preclusion of resale. Specifically, under constrained capacity, if the seller's optimal selling strategy under complete preclusion of resale is premium advance-selling, he can increase profit by switching to partial allowance of resale when the percentage of buyers who arrive early is not too large. If the seller's optimal selling strategy under complete preclusion of resale is discount advance-selling, he can also increase profit by switching to partial allowance of resale in certain cases. We also discuss how the use of online resale venues facilitates partial allowance of resale.

**Keywords:** Resale; Ticket; Advance selling; Online market

## 1. Introduction

In recent years the $200 billion event ticket industry has witnessed a boom in ticket resale facilitated by online consumer-to-consumer markets (Townley 2002). For instance, eBay now offers a major trading category called "Event Tickets" where consumers exchange a large amount of tickets everyday with unprecedented reach and convenience. Business practitioners and researchers are long interested in *how ticket resale affects a monopolistic seller's profit and social welfare*.[1] Recent game-theoretical studies on ticket resale have focused on a two-period setup: an *advance period* followed by a *spot period*. With two periods, the seller can offer tickets at two different posted prices (Rudi, Kapur and Pyke 2001, Lee and Whang 2002, County 2003a, 2003b, Karp and Perloff 2004). Resale has two direct impacts on the seller's profitability. First, a resale market may reduce seller profit in the spot period because some early buyers may decide to resell their own tickets, creating competition with the seller for late buyers. Second, a resale market may increase seller profit in the advance period because the option of resale potentially enhances early buyers' willingness to pay. Besides these two direct effects, the existence or non-existence of a resale market may also affect the seller's sales channel decision (Karp and Perloff 2004), where both decisions eventually affect profit.

Among research papers that support resale from a monopolistic seller's perspective, Deserpa (1994) and Swofford (1999) argue that *uncertainty over buyer valuation and demand* prompts the seller to take the safe route of underpricing, thus opening up the opportunity for resale[2]. Rudi, Kapur and Pyke (2001), and Lee and Whang (2002) assume that *the seller can only sell in the advance period*, and show that the seller can increase his profit in the advance period by allowing resale. Courty (2003a) assumes *ex ante buyer-valuation heterogeneity* ("diehard fans" and "busy professionals"), and shows that resale

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[1] The term "ticket," as studied in the ticket resale literature, refers to an option to consume a single-time consumption good at a fixed consumption time (also called the event time) (Happel and Jennings 2002). Therefore, the category includes more than tickets to entertainment or sports events – a conference registration proof is also a form of ticket. Research on ticket resale is part of the larger literature on ticket selling (see Courty 2000 for an extensive review).

[2] There are some other explanations for underpricing, such as protecting fairness and fostering loyalty that benefit the seller in repeated interactions. See Courty (2003a) for an overview. Since we do not consider repeated interactions, these explanations are out of scope of this paper.

enables the seller to capture value from both types of buyers. Swofford (1999) and Karp and Perloff (2003) assume that *resellers have an ex ante information advantage over the seller* in terms of screening late buyers, thus by allowing resale the seller is able to capitalize on the efficiency gain when more tickets are eventually sold at high prices to high valuation buyers.[3] Note that the applicability of each of these research streams is significantly limited by their core assumptions.

It is then tempting for one to question whether a resale market benefits a monopolistic seller even without underpricing, excluding the seller in the spot period, ex ante buyer-valuation heterogeneity, or ex ante information asymmetry between the seller and resellers. One might think, intuitively, that the answer is positive whenever the seller sells at a high price in the advance period and at a low price in the spot period (*premium advance-selling*[4]), since no arbitrage opportunity is available. However, in a surprising study Courty (2003b) shows that this intuition is wrong. In a model without ex ante buyer heterogeneity and ex ante information asymmetry, and the seller is free to sell in both advance and spot periods, Courty shows that resale is never beneficial for the seller. Even though early buyers cannot arbitrage in premium advance-selling, they will strategically refuse to buy in the advance period, thus annulling the seller's plan for premium advance-selling. Courty's findings suggest that allowing resale is not in a monopolistic seller's best interest, unless specific conditions such as ex ante consumer heterogeneity are satisfied.

In this paper we argue that, even without requiring ex ante buyer-valuation heterogeneity, or ex ante information asymmetry, or limiting the seller's participation in any period, a resale market may still benefit a monopolistic seller *if we consider a resale market as a continuous market in time, and thus divisible*. Except for Courty (2003b), prior research implicitly deems a resale market a unity, and argues either expressly for resale or against it. We consider a third option – it is sometimes possible to divide a continuous resale market into two segments along time, then to allow resale in the first segment and to deny it in the second one. We consider the case where the first segment happens before the seller's spot

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[3] William (1994), in an empirical paper, shows that the existence of resale activities is positively correlated to ticket prices for NFL games. He suggests that this correlation might occur because the resale market provides "true market-clearing price." He also argues that the resale market has information advantage over the seller.
[4] Alternatively, if the seller sells at a low price in the advance period and at a high price in the spot period, it is called *discount advance-selling*.

selling, and the second segment coincides with the spot selling. We call this strategy *partial allowance of resale*, as in contrast to either complete allowance or preclusion of resale.

As the centerpiece of this paper, we show that, *in some cases and under constrained capacity, partial allowance of resale benefits the seller more than either complete allowance or preclusion of resale*. Intuitively, allowing resale in the earlier segment increases early buyers' willingness to pay, while precluding it in the later segment annuls all buyers' incentive to wait and so avoids the strategic waiting problem observed by Courty. Free of the restrictive assumptions of prior research, our results suggest that the benefit of resale for a monopolistic ticket seller is far more general than previously realized.[5]

Under constrained capacity, partial allowance of resale may benefit not only a seller who practices premium advance-selling under complete preclusion of resale, but also one who practices discount advance-selling under complete preclusion of resale. For the latter case, we show by example that this profit increase comes not only from a change in resale policy – from complete preclusion to partial allowance, but also from a corresponding change in selling strategy – from discount to premium advance selling.

A close look at these results reveals a deeper insight, that *partial allowance of resale effectively changes time-based (i.e. based on arrival time) price discrimination to type-based (i.e. based on buyer valuations) price discrimination*. As a result, a seller that is free to allow or preclude partial resale has two price discrimination methods, time- and type-based, at disposal. Whenever type-based price discrimination gives the seller a higher profit, he should allow partial resale.

Partial allowance of resale also has important implications for consumer surplus and social welfare. Most notably, when a seller improves his profit by switching from complete preclusion to partial allowance of resale, it *never* comes at the cost of consumer surplus or social welfare. In fact, if most buyers have low valuations, partial allowance of resale strictly increases consumer surplus and social

[5] Courty (2003b) also considers the possibility of partial resale. Nevertheless, we are the first to find that partial resale may benefit a seller. It turns out that exogenously given capacity constraint, which we consider yet Courty does not, plays a critical role in making partial resale beneficial for the seller.

welfare. Intuitively, the resale of tickets from low-valuation buyers to high-valuation buyers strictly increases social welfare, whereas the improvement is shared by both buyers and the seller if premium advance-selling is the seller's optimal strategy.

Partial allowance of resale can be implemented in various ways. One method is to issue anonymous tickets in an early stage and then require them to be converted later into named ones. We introduce a more implicit way of implementing partial allowance of resale in the online environment called *strategic timing.* With our method, the preclusion of late resale activities is achieved by placing spot selling strategically close to the event time, thereby preventing online resale of spot tickets.

Our research is also related to the advance-selling literature that, like the literature on ticket resale, often uses the two-period posted-price model. (Dana 1998, Shugan and Xie 2000, Xie and Shugan 2001). Specifically, our model is closely related to Xie and Shugan (2001) in that we both consider ex ante homogenous buyers except for different arrival times. Though the primary focus of our research is on resale, our results contribute to the advance-selling literature by demonstrating the value of resale to advance selling.

This paper is organized as follows: Section 2 presents a motivating example and explains the connection between strategic timing and partial allowance of resale. Section 3 presents the model. In Section 4, we derive the equilibrium under partial allowance of resale. We show that partial allowance of resale improves a seller's profit in some cases and discuss managerial and social welfare implications in Section 5. Section 6 concludes the paper.

## 2. A Motivating Example and Strategic Timing

In this section we present a real-life example of ticketing to put our research into a business context and to develop the intuitions behind our model. We then discuss how online resale and strategic timing can help implement partial allowance of resale.

### 2.1. A Motivating Example

The box office at the University of Texas at Austin (UT) sells concert tickets through two major venues. A buyer can either purchase a ticket well in advance (ticket sales usually start two months before a concert) on the box office website or by phone. Alternately, buyers can purchase a ticket on the day of the event at the concert hall. If the concert is popular, it is not unusual to see people line up hours before the spot selling.

For any given concert, buyers always pay more when purchasing online or by phone well in advance, than in person on spot. An advance ticket for one recent concert, for example, costs $41.25, while a spot ticket costs $32.50. This price difference is due to “convenience and handling fees” of totally $8.75 – amounting to a quarter of the ticket’s value – that are imposed when purchasing online or by phone. Yet, the UT box office acknowledged that online sales actually reduce their costs because transaction automation reduces staff workload. Lacking a cost-based reason, the UT box office insists that these fees are just “standard to the industry.”

Obviously, the UT box office is actually practicing premium advance-selling through price-hikes disguised as “standard” fees. What interests us are the two following facts. First, buyers legally and actively resell advance tickets in online consumer-to-consumer marketplaces such as eBay. Second, unlike advance tickets, spot tickets are rarely resold because scalping (on-site resale by consumers) is illegal on campus. Online scalping is considered an off-campus activity and so technically allowed, but spot buyers do not have enough time to accomplish the many steps of an online trade. In other words, the resale market for UT box office tickets is divided into two segments where the first one is active and the second one is closed.

The question of why advance and spot tickets receive different resale treatment is intriguing. From a technological standpoint, banning resale of advance tickets is easy because buyers must submit their names at the time of purchase. The official seller could simply print the buyer’s name on the ticket. However, this may not fit the seller’s best interest. Buyers may hesitate to pay an additional $8.75 two months in advance if they will be stuck with non-transferable tickets. Therefore, allowing resale for advance tickets increases advance buyers’ willingness to pay. This benefit to the seller might be void were on-site resale also allowed. An active spot-resale market provides peace of mind to high-valuation buyers since it insures they will always be able to get tickets from low-valuation buyers who are willing to give up tickets for a small profit over the spot price.

In summary, this real-life example shows the value of partial allowance of resale – allowing resale early and banning it later may provide the right combination of incentives for early buyers to buy early and pay more.

## 2.2. Strategic Timing in Online Resale

It is possible to implement partial allowance of resale in explicit ways, such as by issuing unnamed, transferable tickets at an early stage and then requiring them to be converted to named, non-transferable ones later. Nevertheless, the UT box office never prints names on tickets and so does not depend on any explicit method to achieve partial allowance of resale. Instead, it takes advantage of a Texas law banning physical spot resale, and an often-ignored fact about online consumer-to-consumer markets that effectively blocks online spot resale: the total transaction time for online resale is usually much longer than for traditional resale. Any effort to shorten it may either significantly increase the transaction cost or reduce the chance of a match. A seller who is aware of the long online resale transaction time can effectively implement partial allowance of resale through *strategic timing.* In this method, the seller deliberately sets the spot selling so close to the event that resale in the spot period is essentially impossible.

Though strategic timing is only one way of achieving partial allowance of resale, the growing use of the Internet makes it increasingly important. Strategic timing takes place in a wide range of business practices other than the concert example. As admitted by officials from UT box office that these convenience and handling fees – which cannot be justified on the cost basis – are "standard to the industry," similar practices happen in UT not only for concerts but also for sport events. It also happens in many universities and colleges across the nation. Off campuses, movie theater chains often sell tickets for a higher price (fees included) online than at the gate, and popular tickets are actively traded online well ahead of show time, while trading on spot in front of a movie theatre is often illegal and rarely seen. In addition, higher advance prices often show up as other fees. For instance, fans often need to join an online club and pay a “club membership fee” to secure advance tickets. No such membership is needed if customers buy on spot. (Ordonez 2003).

## 3. The Model

We develop a game-theoretical model for analyzing the potential value of partial allowance of resale. Let there be $\Omega$ buyers (she) and one monopolistic ticket seller (he). Note that $\Omega$ is usually a large number in practice.

**The Seller:** The seller’s marginal cost for producing tickets is a constant, $c > 0$, and he has a capacity of $T$ tickets. Without loss of generality, let $T \leq \Omega$. The seller uses a two-period, posted-price selling strategy. See Figure 1 for the model timeline. This type of ticket selling model is frequently used in the resale and advance-selling literature, and is a good approximation for a large number of ticket-selling practices (Courty 2003a, 2003b, Xie and Shugan 2001). At the start of the advance period, $t_1$, the seller announces the advance price, $p_1$, and makes $S_1$ tickets available for advance selling, where $0 \leq S_1 \leq T$. The advance period lasts until he announces the spot price, $p_2$, and makes the rest of the tickets available at $t_2$ for spot selling. The spot period immediately follows the advance period and lasts until the event (consumption) time, $t_3$. $t_3 > t_2 > t_1 > 0$. The advance period is usually much longer than the spot period in practice (often months versus days or even hours). Thus, we assume $(t_2 - t_1) >> (t_3 - t_2)$. The seller is free to sell either in one single period or in both. For the former case, he can simply make zero tickets available during one period.[6]

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[6] The seller is free to announce $p_2$ well before offering the second round of tickets. However, buyers will not necessarily believe the announced price if the seller can later change it for his own benefit. This is called *the credibility issue,* and it is widely studied in marketing, economics, and finance. The credibility issue is consistent with the fact that, in the ticket market, individual buyers have little power to monitor or enforce a monopolistic

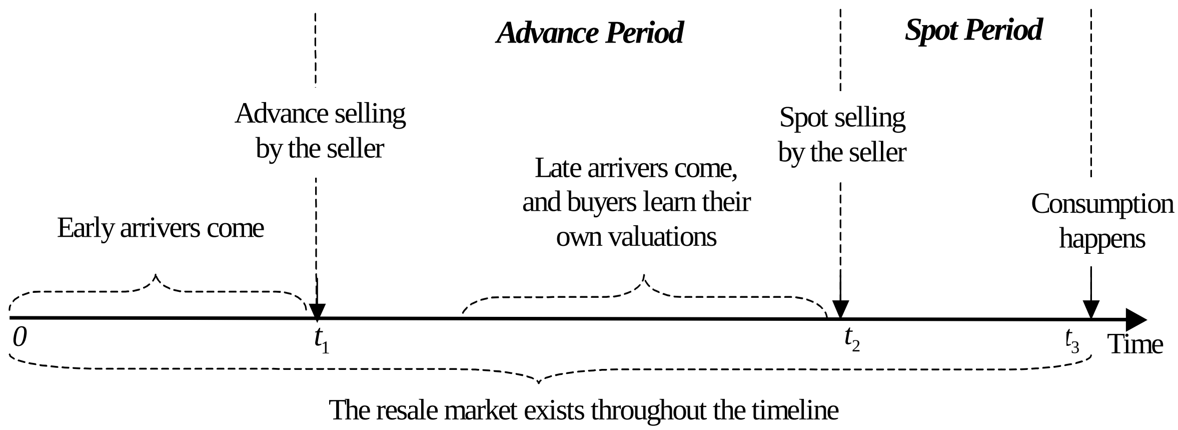


Figure 1. The Model Timeline

**Buyers:** A buyer's ticket valuation can be one of two positive values[7], $H$ or $L$, where $H > L$. Each buyer demands one ticket at most.[8] We consider a case where there is neither ex ante buyer-valuation heterogeneity nor ex ante information asymmetry. All buyers and the seller have the same priori on buyer valuations at the start of the advance period – it is common knowledge that the probability of buyer valuation being $H$ is a constant, $q \in (0,1)$. To avoid discussing trivial cases[9], we assume that $L > c$ and $q < T/\Omega$.

Buyers, however, are differentiated by their arrival times. Suppose $\alpha\Omega$ buyers, called *early arrivers*, arrive before the start of the advance period, where $0 < \alpha < 1$. The remaining $(1-\alpha)\Omega$ buyers, *late arrivers*, arrive in the advance period. Since $(t_2 - t_1) >> (t_3 - t_2)$, if buyer arrivals are smooth, the majority will arrive before or on $t_2$. In light of this, we ignore buyers who arrive after the start of the spot period. If an early arriver buys a ticket at $p_1$ from the seller, we call her an *early buyer*.

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seller's pre-claimed prices (Xie and Shugan 2001, Karp and Perloff 2004). Buyers will only trust the announced $p_2$ from the start of the spot period, when tickets are indeed rolled out at this price.

[7] Strictly speaking, it is the valuation of the associated service provided to the ticket holder.

[8] In the next section, we show that the introduction of ticket brokers/scalpers will not qualitatively change our results.. Brokers and scalpers (see Courty 2003a for a detailed description of these two roles) have zero personal valuation over the tickets and may only buy tickets for arbitrage purposes.

[9] When either $L \leq c$ or $q \geq T/\Omega$, it is apparent that neither selling in both periods nor resale can give the seller a higher profit than simply charging $H$ in the spot period.

Once arriving in the market, a buyer can purchase available tickets and then possibly resell them at any time before the event (consumption) time. Note that a buyer's arrival in the market and her actual purchase are two different concepts because she has the option to wait if she decides to buy or resell later.[10]

Though ex ante, a buyer is uncertain about her valuation of a ticket, she will resolve this uncertainty later in time, such as when she finalizes her schedule. We stipulate that each buyer's uncertainty resolves at a random moment within the advance period, that is, after $t_1$ and before $t_2$. In practice, some buyers may figure out their valuation very early (before or on $t_1$) or very late (on or after $t_2$), but as long as the number of these buyers is very small, it does not qualitatively affect our results.

**The Continuous Resale Market:** A continuous resale market exists throughout the timeline, though it is only useful from $t_1$ to $t_3$. In Section 2, we argued that the resale market is often continuous, and thus divisible by various methods including strategic timing. We consider the case when this resale market can be divided into two segments. The first (second) segment, which we call the *advance-resale (spot-resale) segment*, coincides with the advance (spot) period. Since a transaction-cost-based argument is not the focus of this paper, we assume that a resale segment is perfect if it is allowed.[11]

We call the seller's decision about which resale segment to allow or preclude his *resale policy*. The resale policy consists of four possible scenarios:

1) both resale segments are precluded
2) the advance-resale segment is allowed, while the spot-resale segment is precluded
3) the advance-resale segment is precluded, while the spot-resale segment is allowed
4) both resale segments are allowed

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[10] As a result, low buyer arrival after $t_2$ does not contradict high buyer purchasing after $t_2$. In fact, in the cases we later focus on in this model, a large number of buyers who arrive in the advance period do wait until the spot period to purchase tickets.

[11] A *perfect resale market* is one where matching between buyers and sellers is costless and efficient in the sense that no Pareto-improvement is possible any other way. To the best of our knowledge, the notion of a perfect resale market first appears in Fox (1957).

Prior research deems the resale market a unity and only includes Scenarios 1 (complete preclusion of resale) and 4 (complete allowance of resale). Scenario 3 is new but uninteresting as we will show in Section 5 that it leads to a market equilibrium similar to that under Scenario 4. Therefore, we reserve the term *partial allowance of resale* for Scenario 2. In what follows, we derive the market equilibrium under partial allowance of resale. Then, in Section 5, we compare it to the other scenarios.

## 4. Market Equilibrium Under Partial Allowance of Resale

The advance-resale segment is allowed under partial allowance of resale, while the spot-resale segment is precluded. Let $\pi_1$ denote the seller's profit in the advance period and $\pi_2$ his profit in the spot period. Denote $M_1$ as the number of tickets traded in the advance-resale segment. We use backward induction to solve the model under partial allowance of resale.

### 4.1. The Spot Period

We only need to consider posted-price selling in the spot period because the spot-resale segment is precluded. In the spot period all buyers know their own valuations. Now the spot market is the last chance to purchase a ticket. Consequently, the seller will set his optimal price, $p_2^*$, at either *H* or *L,* depending on whether he wants to serve only high-valuation buyers or all buyers. If $p_2 = H$, then $\pi_2 = (H-c)(q\Omega - qS_1 - M_1)$. If $p_2 = L$, then $\pi_2 = (L-c)(T-S_1)$. Comparing these two numbers yields the following lemma.

**Lemma 1.** *In the spot market: if* $\frac{L-c}{H-c} \leq \frac{q\Omega - qS_1 - M_1}{T - S_1}$*, then* $p_2^* = H$*; otherwise* $p_2^* = L$.

We omit proofs for straightforward lemmas and propositions and put proofs for complex lemmas and propositions in the Appendix.

### 4.2. The Advance-Resale Segment

Note that there are multiple resellers in the advance-resale segment who are all free to negotiate their own resale prices with potential buyers. Nevertheless, in equilibrium, all will pick the same advance-resale price, as shown in the following lemma.

**Lemma 2.** *All resale prices in the advance-resale segment are the same.*

Based on Lemma 2, we can use a time-invariant variable, $p_{r1}$, to denote the resale price in the advance-resale segment.

In the advance-resale segment, there are $(1-q)S_1$ low-valuation early buyers and $q\Omega - qS_1$ high-valuation buyers who have no tickets. If $(1-q)S_1 > q\Omega - qS_1$ (or equivalently $S_1 > q\Omega$) then supply in the advance-resale segment is larger than the demand even if the seller does not sell any tickets in the spot period. Consequently, the equilibrium advance-resale price, $p_{r1}^*$, will be driven down to *L* because of competition among resellers. Since buyers are certain that there are enough tickets in the resale market at price *L*, they will not pay more than *L* during the advance period. This is equivalent to the case where the seller only spot sells at price *L*.

Therefore, a necessary condition for the seller to advance sell is $S_1 \le q\Omega$, under which resellers can command a high price as long as it does not drive high-valuation buyers away.

**Lemma 3.** *Suppose that* $S_1 \le q\Omega$. *The number of tickets traded in the advance-resale segment,* $M_1$, *is equal to* $(1-q)S_1$. *Moreover, if* $\frac{L-c}{H-c} \le \frac{q\Omega - S_1}{T-S_1}$, *equilibrium advance-resale price is* $p_{r1}^* = H$.

*Otherwise* $p_{r1}^* = L + \frac{\Omega - T}{\Omega - S_1}(H-L)$.

Intuitively, resellers should offer an advance-resale price that makes high-valuation buyers indifferent about buying in the advance-resale segment or in the spot period. The preclusion of the spot-resale market makes high-valuation buyers uncertain about a ticket in the spot period, so they will pay an insurance premium, $\frac{\Omega - T}{\Omega - S_1}(H - L)$, in the advance-resale segment.

When the number of resellers equals the number of high-valuation buyers without tickets, $(1-q)S_1 = q\Omega - qS_1$, a reseller and a high-valuation buyer may settle on any incentive compatible price. Thus, advance-resale prices can be uncertain. Nevertheless, in the proof of Lemma 3, we implicitly assumed that resellers have the market power in that they ask for the highest possible price. Such an assumption simplifies our analysis without loss of generality: Since the resale price is crucial for supporting the seller's advance price, as shortly shown in Lemma 4, the seller can ensure that resellers have clear market power over high-valuation buyers by selecting $S_1$ such that it is only infinitesimally smaller than $q\Omega$.

### 4.3. The Advance Selling

Now we need only consider the case where $S_1 \le q\Omega$. A buyer's expected surplus from advance buying is *ESA* = $q(H - p_1) + (1-q)(p_{r1} - p_1)$, and her expected surplus from waiting is *ESW* = $q(H - p_{r1})$. To induce buyers to advance purchase, the seller must pick $p_1$ such that $ESA \ge ESW$. Therefore, to maximize his profit, the seller will pick $p_1$ such that $ESA = ESW$.

**Lemma 4.** *In the advance period, the seller's optimal price is* $p_1^* = p_{r1}$.

Now we can compute the seller's total profit for any $S_1$.

**Lemma 5.** *Given any* $S_1$ *such that* $S_1 \le q\Omega$. *If* $\frac{L-c}{H-c} > \frac{q\Omega - S_1}{T - S_1}$, *then* $p_2^* = L$,

$p_1^* = p_{r1}^* = L + \frac{\Omega - T}{\Omega - S_1}(H-L)$, *and the seller's total profit is* $\pi^* = T(L-c) + S_1 \frac{\Omega - T}{\Omega - S_1}(H-L)$;

*otherwise* $p_1^* = p_{r1}^* = p_2^* = H$, *and the seller's total profit is* $\pi^* = q\Omega(H-c)$.

We noted above that if $S_1 > q\Omega$, then $\pi = T(L-c)$. From Lemma 5 we know that by reducing $S_1$ to less than $q\Omega$, the seller's profit will change to either $T(L-c) + S_1 \frac{\Omega - T}{\Omega - S_1}(H-L)$ or $q\Omega(H-c)$ depending on how large $\frac{L-c}{H-c}$ is. Apparently, $T(L-c) + S_1 \frac{\Omega - T}{\Omega - S_1}(H-L)$ is no less than $T(L-c)$ ,and when $\frac{L-c}{H-c} \le \frac{q\Omega - S_1}{T - S_1}$, $q\Omega(H-c)$ is also no less than $T(L-c)$. Therefore, we have the following lemma.

**Lemma 6.** *The optimal quantity of tickets for advance selling,* $S_1^*$, *always satisfies* $S_1^* \le q\Omega$.

The number of advance tickets sold cannot be greater than the number of early arrivals, i.e. $S_1^* \le \alpha\Omega$ , therefore $S_1^* \le \min\{q\Omega, \alpha\Omega\}$. If $S_1^* = 0$, it means the seller will not advance sell.

### 4.4. The Seller's Optimal Selling Strategy Under Partial Allowance of Resale

We can now determine the seller's optimal selling strategy. The seller can either pick $S_1^* = 0$, spot-only selling, or $S_1^* > 0$, selling in both periods. If he chooses spot-only selling, he can either sell all tickets at *L* or sell only to high-valuation buyers at *H*. If he chooses advance selling, he must pick the optimal price and number of tickets for advance selling according to Lemma 5. By comparing the profits from all these alternatives, we have Proposition 1.

**Proposition 1.** *Table 1 shows the market equilibrium under partial allowance of resale*.

| Condition | Equilibrium Strategy | Seller Profit |
|---|---|---|
| **(I)** $q \le \alpha$ and $\frac{L-c}{H-c} \le q$ | Spot-only selling $p_1^* = p_{r1}^* = p_2^* = H$ | $q\Omega(H-c)$ |
| **(II)** $q \le \alpha$ and $\frac{L-c}{H-c} > q$ | Premium-advance selling $p_2^* = L$, $S_1^* = q\Omega$, $p_1^* = p_{r1}^* = L + \frac{\Omega - T}{(1-q)\Omega}(H-L)$ | $T(L-c) + \frac{q}{1-q}(\Omega - T)(H-L)$ |
| **(III)** $T/\Omega > q > \alpha$ and $\frac{L-c}{H-c} \le \alpha + (q-\alpha)\frac{1-\alpha}{T/\Omega - \alpha}$ | Spot-only selling $p_1^* = p_{r1}^* = p_2^* = H$ | $q\Omega(H-c)$ |
| **(IV)** $T/\Omega > q > \alpha$ and $\frac{L-c}{H-c} > \alpha + (q-\alpha)\frac{1-\alpha}{T/\Omega - \alpha}$, | Premium-advance selling $p_2^* = L$, $S_1^* = \alpha\Omega$, $p_1^* = p_{r1}^* = L + \frac{\Omega - T}{(1-\alpha)\Omega}(H-L)$ | $T(L-c) + \frac{\alpha}{1-\alpha}(\Omega - T)(H-L)$ |

Table 1. Equilibrium Strategies Under Partial Allowance of Resale

In Proposition 1, we consider only $q < T/\Omega$. When $q \ge T/\Omega$, it is apparent that spot-only selling at *H* gives the seller the highest possible profit. Note that Cases III and IV exist if and only if $T/\Omega > \alpha$. Also note that when we say $S_1^* = q\Omega$, as in Case II, the seller may actually set an advance-selling quantity of $q\Omega$ minus 1 to avoid any uncertainty in advance-resale price. This impacts the seller's profit only infinitesimally when $\Omega$ is large.

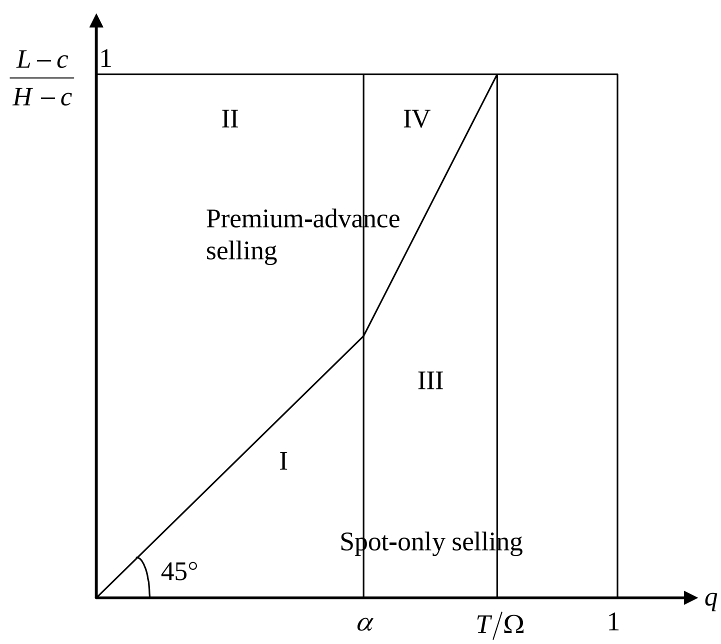


Figure 2. Optimal Selling Strategy Under Partial Allowance of Resale

Areas I-IV in Figure 2 correspond to the four cases in Proposition 1. The seller's optimal profit function is continuous on the border of any two adjacent areas.

Under partial allowance of resale, the seller's decision about allowing premium-advance selling depends on two critical ratios: the ratio of net low-value to net high-value, $\frac{L-c}{H-c}$ or the *value ratio*, and the ratio of high-valuation buyers to all buyers, $q$ or the *type ratio*. Figure 2 shows that premium-advance selling requires a larger value ratio than type ratio.[12] Intuitively, a larger value ratio and a smaller type ratio make low-valuation buyers more attractive to the seller. This gives the seller more incentive to cater to low-valuation buyers by spot selling at $L$, which leads to premium-advance selling if the number of tickets, $T$, is limited.

**Corollary 1.** *Under partial allowance of resale, discount-advance selling is impossible in equilibrium.*

Corollary 1 directly follows Proposition 1. If the advance-resale segment exists, the seller is discouraged from discount-advance selling because it gives resellers a guaranteed, highly-profitable arbitrage opportunity and reduces the seller's profit.

However, lack of discount-advance selling under partial allowance of resale does *not* imply that resale cannot improve the seller's profit when he uses discount-advance selling under complete preclusion of resale. We will expand on this point in Section 5.

### 4.5. The Impact of Brokers/Scalpers

[12] How much larger the value ratio should be than the type ratio depends on two additional ratios. First, the ratio of the number of early-arrivers to the number of all buyers, $\alpha$ or the *arrival ratio* (conditional on $S_1^* = \alpha\Omega$). Second, the ratio of the number of available spot-period tickets to the number of buyers without tickets at the beginning of the spot period, $\frac{T/\Omega-\alpha}{1-\alpha}$ or the *spot ticket-availability index*. Additionally, we call $T/\Omega$ the *overall ticket-availability index.*

We did not consider brokers and scalpers above. As discussed in Courty (2003a), brokers and scalpers buy tickets for the sole purpose of resale. Their own valuation of a ticket is zero. Their only difference is that brokering is a legitimate business while scalping is often illegal.

Consider adding some brokers/scalpers into our model. Since the spot-resale segment is precluded, the only potential arbitrage opportunity for brokers/scalpers is to buy tickets during the advance period and resell them during the advance-resale segment. However, as in equilibrium $p_1 = p_{r1}$, this scenario will earn brokers/scalpers zero profit. Moreover, their existence does not affect ticket allocation at the end of the advance-resale segment, because all tickets held by low-valuation buyers are eventually transferred to high-valuation buyers.

## 5. Optimal Resale Policies

Whenever the continuous resale market exists and is divisible, a seller's decision variables include his resale policy and selling strategy. In this section, we focus on analyzing the seller's equilibrium resale policy, with an emphasis on the implications of partial allowance of resale. We also address some managerial, consumer surplus, and social welfare implications.

### 5.1. Benchmark Scenarios

We first derive the market equilibria under the other three scenarios (see the end of Section 3). Without loss of generality, we assume that if the seller's profits from advance selling are equal to or less than the profits from spot-only selling, he will opt for spot-only selling.

**Proposition 2.** *If the spot-resale segment is allowed (as in Scenarios 3 and 4), the seller's optimal selling strategy is always spot-only selling. This is shown in Table 2.*

| Condition | Equilibrium Strategy | Seller Profit |
|---|---|---|
| (I) $q \le \frac{T}{\Omega} \bullet \frac{L-c}{H-c}$ | Spot-only selling $p_2^* = L$ | $T(L-c)$ |
| (II) $q > \frac{T}{\Omega} \bullet \frac{L-c}{H-c}$ | Spot-only selling $p_2^* = H$ | $q\Omega(H-c)$ |

Table 2. Market Equilibrium When the Spot-Resale Segment Is Allowed

This result is surprising because Scenarios 3 and 4 use different resale policies. When the spot-resale segment is allowed, high-valuation buyers have the option of waiting until the last-minute and then buying from low-valuation resellers. This causes a strategic waiting problem that voids the seller's plan to profit from advance selling and resale.

**Proposition 3.** *Table 3 shows the market equilibrium under complete preclusion of resale.* [13]

| Condition | Equilibrium Strategy | Seller Profit |
|---|---|---|
| (I) $q < \frac{T/\Omega - \alpha}{1-\alpha} \bullet \frac{L-c}{H-c}$ | Premium advance selling $p_2^* = L$, $S_1^* = \alpha\Omega$, $p_1^* = L + q\frac{\Omega - T}{(1-\alpha)\Omega}(H-L)$ | $T(L-c) + q\frac{\alpha}{1-\alpha}(\Omega - T)(H-L)$ |
| (II) $q \le \frac{T/\Omega - \alpha}{1-\alpha}$ and $q \ge \frac{T/\Omega - \alpha}{1-\alpha} \bullet \frac{L-c}{H-c}$, | Discount advance selling $p_2^* = H$, $S_1^* = \alpha\Omega$, $p_1^* = L + q(H-L)$, | $q\Omega(H-c) + (1-q)\alpha\Omega(L-c)$ |
| (III) $T/\Omega > q > \frac{T/\Omega - \alpha}{1-\alpha}$, | Discount advance selling $p_2^* = H$, $S_1^* = \frac{T - q\Omega}{1-q}$, $p_1^* = L + q(H-L)$, | $T(L-c) + q\Omega(H-L)$ |

Table 3. Market Equilibrium Under Complete Preclusion of Resale

In Table 3 we consider only the case where $q < T/\Omega$. When $q \ge T/\Omega$, spot-only selling at $H$ gives the seller the highest possible profit. In addition, Cases I and II exist if and only if $T/\Omega > \alpha$ since $q$ is a positive number.

Proposition 3 is a generalization of Theorem 7 from Xie and Shugan 2001. In their model, half buyers arrive for advance selling. In our Proposition 3, any arbitrary portion of buyers, $\alpha$, may arrive for

[13] The proof of Proposition 3 is lengthy and uses backward induction similar to the analysis in Section 4,so we put it in a separate technical appendix called "Proof of Proposition 3.pdf."

advance selling. This distinction is important because we will later show that the value of partial allowance of resale to the seller depends on $\alpha$ . Proposition 3 is illustrated in Figure 3. Areas I-III correspond to the three cases in Proposition 3.

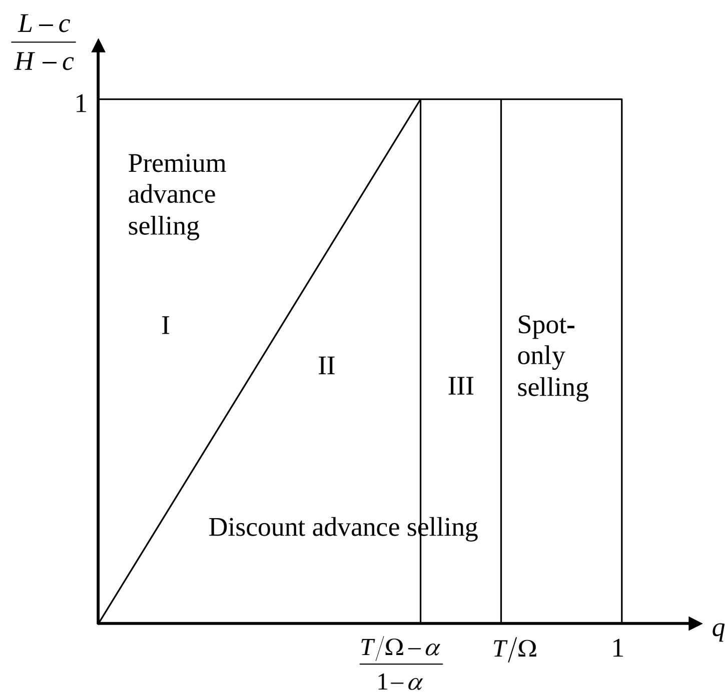


Figure 3. Optimal Selling Strategy Under Complete Preclusion of Resale

**5.2. Optimal Resale Policies**

We now summarize the seller's optimal resale policy with three propositions.

**Proposition 4.** *The seller is never better off by allowing the spot-resale segment, compared with precluding this segment.*

In other words, Scenarios 1 and 2 dominate Scenarios 3 and 4. This result comes directly from a comparison of Proposition 2 and Propositions 1 and 3. Consequently, the seller's optimal resale policy, except in some trivial cases, is either complete preclusion of resale or partial allowance of resale.[14] This result is consistent with the findings in Courty (2003b). What new here is that our result points out it is the last segment of the continuous resale market – not the whole resale market – that results in the strategic waiting problem that harms a seller's profit.

Hereafter, we consider only the case where the spot-resale segment is precluded. This leaves Scenarios 1 (complete preclusion of resale) and 2 (partial allowance of resale).

**Proposition 5.** *Without exogenously given capacity constraint, i.e.* $T = \Omega$*, the seller is never better off by allowing partial or complete resale, compared with precluding resale.*

This proposition comes directly from a comparison of Tables 1 and 3 after substituting $T$ with $\Omega$. Intuitively, when there is no capacity constraint, high-valuation buyers face no risk in terms of eventually securing a ticket, thus they will not pay any risk premium in the resale market, which in turn makes it impossible for the seller to charge a very high advance price.

**Proposition 6.** *Assume that* $T < \Omega$*, i.e. there is an exogenously given capacity constraint. In a comparison between partial allowance of resale and complete preclusion of resale:*

*(I)* *If* $q < \frac{T/\Omega - \alpha}{1-\alpha} \bullet \frac{L-c}{H-c}$*, the seller always chooses premium advance-selling under either complete preclusion of resale or partial allowance of resale. The seller makes more profit under partial allowance of resale if and only if* $\alpha < 1/(2-q)$.

*(II)* *If* $\frac{T/\Omega - \alpha}{1-\alpha} \bullet \frac{L-c}{H-c} \le q \le \min\left\{\frac{L-c}{H-c}, \frac{T/\Omega - \alpha}{1-\alpha} \bullet \frac{L-c}{H-c} + \frac{\alpha(1-T/\Omega)}{1-\alpha}, \frac{T/\Omega - \alpha}{1-\alpha}\right\}$*, the seller chooses discount advance-selling under complete preclusion of resale and premium*

[14] Strictly speaking, in some trivial cases where the seller's optimal selling strategy is spot-only selling under all four resale scenarios, his resale policy does not matter and thus all four resale scenarios are optimal.

*advance-selling under partial allowance of resale. The seller's resale policy depends on which of the following two profits is larger:*

- *profit under partial allowance of resale:* $T(L-c)+\min\left\{\frac{q}{1-q},\frac{\alpha}{1-\alpha}\right\}\bullet(\Omega-T)(H-L)$,
- *profit under complete preclusion of resale:* $q\Omega(H-c)+(1-q)\alpha\Omega(L-c)$.

*(III)* *If* $\min\left\{\frac{L-c}{H-c},\frac{T/\Omega-\alpha}{1-\alpha}\bullet\frac{L-c}{H-c}+\frac{\alpha}{1-\alpha}(1-T/\Omega),\frac{T/\Omega-\alpha}{1-\alpha}\right\}<q<T/\Omega$*, the seller chooses discount advance-selling under complete preclusion of resale and spot-only selling under partial allowance of resale. The seller always makes more profit under complete preclusion of resale.*

*(IV)* *If* $q\geq T/\Omega$*, the seller always chooses spot-only selling.*

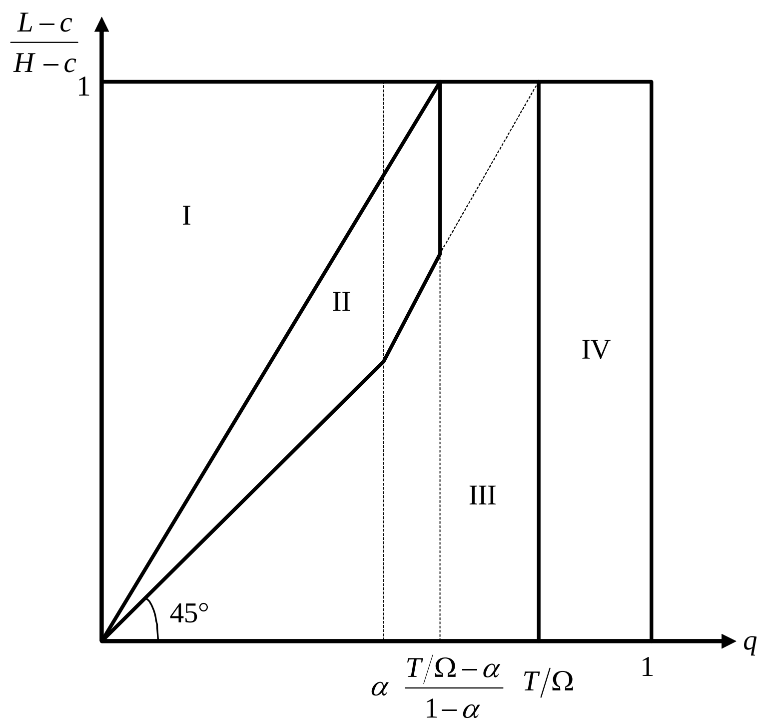


Figure 4. Comparing Partial Allowance of Resale and Complete Preclusion of Resale

The seller's optimal resale policy is uniquely determined by four factors: type ratio $q$, value ratio $\frac{L-c}{H-c}$, arrival ratio $\alpha$, and the overall ticket-availability index $T/\Omega$. Areas I–IV in Figure 4 (separated by solid lines) corresponds to Cases I–IV in Proposition 6. Areas I and II exist if and only if $T/\Omega > \alpha$, and the shape of Areas II and III may change depending on which of $\frac{T/\Omega - \alpha}{1-\alpha}$ and $\alpha$ is larger.

Although complete allowance of resale cannot benefit the seller, Proposition 6 shows that partial allowance of resale might. This important proposition gives clear managerial guidelines for the adoption of partial allowance of resale, which we will discuss further in the next subsection. To summarize: in Figure 4, Area I, the seller should adopt partial allowance of resale when the percentage of early arrivers is relatively small. In Area II, a simple and general criterion for the adoption of partial allowance of resale is not available, but a seller can always judge by comparing the two profit formulas.

### 5.3. Managerial Implications

In this subsection, we organize the managerial implications of Propositions 4-6 into five results and focus on the benefits of partial allowance of resale for the seller. We use complete preclusion of resale as the benchmark and ask the question "can the seller benefit by switching from complete preclusion of resale to complete or partial allowance of resale?"

**Result 1.** *Complete allowance of resale never benefits a ticket seller*.

If the seller uses discount advance-selling, ticket sales in the advance period will cannibalize sales in the spot period since early buyers will compete with the seller. If the seller uses premium advance-selling, early arrivers will strategically wait since they can always get tickets in the resale market, thereby annulling the seller's attempt to sell at a high price early.

**Result 2.** *Partial allowance of resale can benefit a ticket seller in some cases, and it is crucial to allow only the advance-resale segment.*

A seller should be aware of the option of partial allowance of resale, and of the two effects it creates that jointly support his profit gain. First, allowing resale at an early stage effectively provides insurance to early buyers and increases buyer valuation in that stage. Second, precluding resale at a later stage prevents early arrivers from strategically waiting until the spot sale.

**Result 3.** *A necessary condition for the seller to allow partial resale is an exogenously given capacity constraint.*

One key reason of why resale might help the seller is that, under premium advance-selling, high-valuation buyers are willing to pay a positive risk-premium in the resale market to secure a ticket, which in turn enables the seller to charge a high advance price. Exogenously given capacity constraint is a necessary condition for this positive risk-premium: without this constraint, under premium advance-selling all buyers can eventually get a ticket at a low spot price.

**Result 4.** *If the seller's optimal selling strategy under complete preclusion of resale is premium advance-selling, he can increase profit by switching to partial allowance of resale if and only if the percentage of early arrivers is not too large (i.e.* $\alpha < 1/(2-q)$ *).*

This result is a non-mathematical rewrite of Proposition 6 (I). Note that the percentage requirement of early arrivers for Result 3 is not very restrictive. For instance, any $\alpha \le 1/2$ satisfies this requirement. In other words, a seller practicing premium advance-selling under complete allowance of resale will always find himself better off by switching to partial allowance of resale if the majority of buyers arrive late.

In order to understand this result, it is important to see what market dynamics are similar under these two resale scenarios, and what are different. We first list similarities. The seller chooses premium advance selling when the value ratio, $\frac{L-c}{H-c}$, is relatively large and the type ratio, $q$, is relatively small (see Area I in Figure 4). Under these two conditions low-valuation buyers are important for the seller, who will then cater to their demand by setting a low spot price at $L$. As a result, the seller's choice of resale policy depends on how much profit he can make in the advance period. Under premium advance-selling, a high price in the advance period serves as a price-discrimination device that enables the seller to make a higher per ticket profit than in the spot period.

Nevertheless, buyers been discriminated are *different* under the two resale scenarios. Under complete preclusion of resale *all early arrivers*, no matter if their valuation is high or low, are discriminated. Under partial allowance of resale *all high-valuation buyers* (or most of them if $q > \alpha$), no matter if they are early or late, are discriminated.[15] In other words, *the allowance of resale activities in the advance period changes time-based price discrimination to type-based price discrimination*.

This change of targeting population in price discrimination makes partial allowance of resale more favorable when the percentage of high-valuation buyers, $q$, is large enough.[16] If $q$ increases and as long as it is no greater than $\alpha$, both the per ticket profit in the advance period and the size of the discriminated population (all high-valuation buyers) increases. Under complete preclusion of resale, however, the size of the discriminated population remains constant even when $q$ increases.

**Result 5.** *In certain cases, the seller can increase profit by switching to partial allowance of resale even if his optimal selling strategy under complete preclusion of resale is discount advance-selling.*

Result 4 is surprising since it refutes the conventional wisdom that a seller who is practicing discount advance-selling cannot benefit from allowing resale activities because of the existence of early buyer arbitrage opportunities. Key to this result is the fact that when the seller changes his resale policy from

[15] For instance, in Case II of Proposition 1, a high-valuation buyer either pays the seller's advance price or an equal resale price, no matter if she comes in early or late.
[16] Note that $\alpha < 1/(2-q)$ can be rewritten as $q > 2 - 1/\alpha$.

complete preclusion to partial allowance, in equilibrium he also needs to change his selling strategy since discount advance-selling is no longer optimal (recall Corollary 1). This change of selling strategy solves the arbitrage issue.

Because of the simultaneous change of resale policy and selling strategy, we cannot give simple criteria for when partial allowance of resale benefits the seller more than complete preclusion of resale. Nevertheless, for any set of given parameters the seller can simply compute the profits under both resale scenarios and then select the best one. Below we give an example of Result 5.

**Example.** Let $\alpha = q = 0.15$, $(L-c)/(H-c) = 0.36$, and $T/\Omega = 0.5$. This set of parameters satisfies the condition in Proposition 5 (II). Therefore, under complete preclusion of resale, the seller will adopt discount advance-selling, which leads to a profit of $0.1959\Omega(H-c)$. Under partial allowance of resale, the seller will adopt premium advance-selling, which leads to a profit of $0.2365\Omega(H-c)$. By switching from complete preclusion of resale to partial allowance of resale, the seller's profit increases by 20.7%.

### 5.4. Consumer Surplus and Social Welfare

In some cases, partial allowance of resale benefits more than just the monopolistic ticket seller. In this subsection, we show how it also increases consumer surplus and social welfare. We first establish the following lemma.

**Lemma 7.** *Given that the seller advance sells, all early low-valuation buyers suffer a net loss ex post under complete preclusion of resale, but no buyer suffers a net loss ex post under partial allowance of resale.*

This revealing lemma stems from the fact that the price early buyers pay is strictly higher than $L$ under either premium or discount advance selling. Under complete preclusion of resale, early buyers are stuck with a ticket, thus one with a low-valuation ends up paying more than what the ticket is worth.

Under partial preclusion of resale, however, they can resell the ticket to avoid net loss. This leads to a more efficient resource allocation among buyers.

**Proposition 7.** *Compared with the case under complete preclusion of resale, consumer surplus is never lower under partial allowance of resale. Moreover, if the seller's optimal resale policy is partial allowance of resale, consumer surplus is strictly higher except for* $\alpha \le q \le \frac{T/\Omega - \alpha}{1-\alpha} \bullet \frac{L-c}{H-c}$ .

Proposition 7 gives a strong result that, whenever partial allowance of resale is the seller's optimal resale policy, consumer surplus is never hurt. In fact, it often leads to a win-win situation for both the seller and buyers. The win-win situation holds when, for instance, most buyers have low valuations ( $q < \alpha$ ).

## 6. Conclusion

The conventional wisdom is that a resale market is either entirely good or entirely bad for monopolistic ticket sellers. The primary goal of this research is to emphasize the existence and value of a third option: often, a resale market is continuous and divisible. We show that, while the later segment of a resale market hinders a seller's attempt at price discrimination in advance-selling because of the strategic waiting problem, the earlier segment actually benefits the seller by raising buyers' willingness to pay. As a result, partial allowance of resale, which is a combination of allowing early resale and precluding late resale, increases the seller's profit in many cases. This finding suggests that the value of resale to a ticket seller is far more general than previously realized. Our results also give clear guidelines for when the seller should adopt partial allowance of resale to increase profit. Finally, the seller's gain from partial allowance of resale often benefits consumer surplus or social welfare.

Our research also challenges the technology advancement-based argument against resale. We question why technology has to be used *against* resale as is widely observed in business practice. Our discussion of strategic timing in online markets and, specifically, the real-life example of the UT box office shows that technological advancements can be used to manipulate resale activities in more delicate ways. This allows both buyers and sellers to harvest efficiency gains from resale activities while avoiding the strategic waiting problem that may defeat the seller's two-period price discrimination strategy.

**Appendix**

**Proof of Lemma 2:** If there are more low-valuation ticket resellers in the advance-resale segment than high-valuation buyers without a ticket, then competition on the resellers' side will drive resale down to the time-invariant price $L$.

Otherwise, denote the negotiated price of the *n*'th ticket resold in the advance-resale segment as $P_{r1}(n)$, where $1 \le n \le M_1$. For convenience, we call the reseller of the *n*'th ticket the *n*'th reseller. Now we want to show that $P_{r1}(n) = P_{r1}(M_1)$ holds true for any $n < M_1$. $P_{r1}(M_1)$ is the price of the last ticket resold in the advance-resale segment, and $P_{r1}(M_1)$ is determined such that a high-valuation buyer is indifferent about buying from the $M_1$' th reseller or waiting for the monopolistic seller's spot selling.

(i) If for some $n < M_1$ we have $P_{r1}(n) < P_{r1}(M_1)$, then the best strategy for the *n*'th reseller is to refuse to sell until the price is high because there are always high-valuation buyers without a ticket.

(ii) If for some $n < M_1$ we have $P_{r1}(n) > P_{r1}(M_1)$, then a high-valuation buyer without a ticket should refuse to buy since she will be better off waiting for the spot selling.

**Proof of Lemma 3:** Since $(1-q)S_1 \le q\Omega - qS_1$, a reseller will always be able to find a buyer if he offers a low enough price. Because the advance-resale market provides a reseller with the only chance to resell, he has the incentive to do so, thus $M_1 = (1-q)S_1$. The highest price an advance reseller should offer is one that makes a buyer indifferent about buying from him or from a reseller in the spot market.

Apparently, when $\frac{L-c}{H-c} \leq \frac{q\Omega - S_1}{T-S_1}$, all buyers will expect the seller to choose $p_2^* = H$ in the spot market (recall Lemma 1), therefore $p_{r1}^* = H$. When $\frac{L-c}{H-c} > \frac{q\Omega - S_1}{T-S_1}$, all buyers will expect the seller to choose $p_2^* = L$ in the spot market. Since all buyers will try to secure a ticket in the spot market, a high-valuation buyer only has $\frac{T-S_1}{\Omega - S_1}$ chance to get one. From $H - p_{r1}^* \geq \frac{T-S_1}{\Omega - S_1}(H-L)$, we have $p_{r1}^* = L + \frac{\Omega - T}{\Omega - S_1}(H-L)$.

**Proof of Proposition 1:** From Lemma 5 we have: if $\frac{L-c}{H-c} > \frac{q\Omega - S_1}{T-S_1}$, or equivalently if

$$S_1 > \frac{q\Omega(H-c) - T(L-c)}{H-L} \qquad \ldots\text{(a)}$$

, then $\pi^* = T(L-c) + S_1 \frac{\Omega - T}{\Omega - S_1}(H-L)$, which is an increasing function of $S_1$. Recall we have $S_1^* \leq \min\{q\Omega, \alpha\Omega\}$.

If $q \leq \alpha$, then $S_1^* \leq q\Omega$. It can be verified that $q\Omega > \frac{q\Omega(H-c) - T(L-c)}{H-L}$. Given that when (a) holds the profit is an increasing function of $S_1$, in equilibrium we have either $S_1 = q\Omega$ or $S_1 \leq \frac{q\Omega(H-c) - T(L-c)}{H-L}$. In the former case, $\pi^* = T(L-c) + \frac{q}{1-q}(\Omega - T)(H-L)$, in the latter case $\pi^* = q\Omega(H-c)$. Comparing these two possible profits yeilds Cases I and II.

The proof is similar when $q > \alpha$, only that now it is possible to have $\alpha\Omega \leq \frac{q\Omega(H-c) - T(L-c)}{H-L}$, under which $S_1 \leq \frac{q\Omega(H-c) - T(L-c)}{H-L}$ is always true.

**Proof of Proposition 2:** The seller should only set spot price $p_2$ as *H* or *L* to maximize profit during spot selling. At any $p_2$ between *H* and *L*, the seller faces the same demand as at $p_2 = H$, but gains less profit from each ticket.

(I) All *T* tickets will be sold out if $p_2 = L$. Since $q\Omega < T$, all high-valuation buyers may purchase tickets from either the seller at $p_2 = L$ or from the spot-resale market at $p_{r2} = L$. Therefore, the advance selling price $p_1$ can only be *L*. Thus, the seller's total profit is $T(L-c)$, which is less than or equal to his profit without advance selling.

(II) No low-valuation buyers will buy tickets from the seller in spot period if $p_2 = H$ because resellers will set the resale price lower than *H*.

If $S_1 > q\Omega$, there will be more resellers than high-valuation buyers in the spot period. The spot-resale price $p_{r2}$ will converge to *L* and no one will buy from the seller in the spot period. Furthermore, since high-valuation buyers will always get tickets at *L* from the resale market, they can only accept an advance selling price, $p_1$, that is no higher than *L*. Thus, the seller's total profit is $S_1(L-c)$, which is less than that from spot-only selling.

If $S_1 \le q\Omega$, all low-valuation early buyers will resell their tickets to the high-valuation buyers in either the advance-resale market or the spot-resale market. Thus, in the spot period the seller can sell no more than $q\Omega - S_1$ tickets as long as $p_2 > L$. Therefore, the seller's total profit is less than or equal to $q\Omega(H-c)$, which is the seller's profit without advance selling.

**Proof of Proposition 6:** To decide the optimal choice, we need to compare the profits in Tables 1 and 3 for all possible overlapping situations. In other words, pick any point in the $q - \frac{L-c}{H-c}$ space, check which area it belongs to in Figures 2 and 3, find the respective profits in Tables 1 and 3, and compare them. For convenience, we use AI, AII, AIII, and AIV to represent cases I, II, III, and IV in Table 1, and

BI, BII, and BIII to represent cases I, II, and III in Table 3. To further simplify, we write $AX \geq BY$ if we pick any point from the intersection of cases AX and BY (X=I~IV and Y=I~III), and find that seller profit on this point in Table 1 is always higher than or equal to that in Table 3. Similarly we can have other inequalities between cases.

BI may overlap with AII and AIV. Comparing BI and AII: $BI \leq AII$ iff

$$\pi_{AII} = T(L-c) + \frac{q}{1-q}(\Omega - T)(H-L) \geq \pi_{BI} = T(L-c) + q\frac{\alpha}{1-\alpha}(\Omega - T)(H-L)$$, or

$\frac{1}{1-q} \geq \frac{\alpha}{1-\alpha}$, or $\alpha < \frac{1}{2-q}$. If BI overlaps with AIV, then $BI \leq AIV$ since

$$\pi_{AIV} = T(L-c) + \frac{\alpha}{1-\alpha}(\Omega - T)(H-L) > \pi_{BI} = T(L-c) + q\frac{\alpha}{1-\alpha}(\Omega - T)(H-L)$$. Also note that

$\alpha \leq q$ in the joint area of BI and AIV. Since $q < \frac{1}{2-q}$, $\alpha < \frac{1}{2-q}$ is always true here.

BII may overlap with AI, AII, AIII and AIV. Proposition 3 proved that discount advance selling is more profitable that spot only selling in case BII. Thus, $BII > AI$ and $BII > AIII$. There is no simple formula for determining whether the profit in AII or AIV is larger than that in BII.

BIII may overlap with AI, AII, AIII and AIV. By Proposition 3, $BIII > AI$ and $BIII > AIII$. Given $T/\Omega > q$, we also have $BIII > AII$ since,

$$\pi_{AII} = T(L-c) + \frac{q}{1-q}(\Omega - T)(H-L) < \pi_{BIII} = T(L-c) + q\Omega(H-L)$$. In AIV $q > \alpha$,

$$\pi_{AIV} = T(L-c) + \frac{\alpha}{1-\alpha}(\Omega - T)(H-L) < T(L-c) + \frac{q}{1-q}(\Omega - T)(H-L) < \pi_{BIII} = T(L-c) + q\Omega(H-L)$$,

thus we have $BIII > AIV$.

**Proof of Proposition 7:** We use AI, AII, AIII and AIV to represent cases I to IV in Table 1, and BI, BII and BIII to represent cases I to III in Table 3.

In AI and AIII, consumer surplus is zero because the seller only sells tickets at H to high-valuation buyers. In AII, consumer surplus is $CS_{AII} = q\Omega(H - L)\frac{T - q\Omega}{\Omega - q\Omega}$. In AIV, consumer surplus is

$$CS_{AIV} = q\Omega(H - L)\frac{T - \alpha\Omega}{\Omega - \alpha\Omega}.$$

In BII and BIII consumer surplus is also zero. Some early buyers may have high-valuation and will gain a surplus of $qS_1(H - (L + q(H - L))) = q(1 - q)S_1(H - L)$, but the others have low-valuation and will get a negative surplus of $(1 - q)S_1(-q(H - L)) = -q(1 - q)S_1(H - L)$. Therefore, the total surplus under discount advance-selling is zero. BI may overlap with AII and AIV. In BI, consumer surplus is $CS_{BI} = q\Omega(H - L)\frac{T - \alpha\Omega}{\Omega - \alpha\Omega} = CS_{AIV}$. Also $CS_{BI} < CS_{AII}$ since $q < \alpha$ in AII. Proposition 6 follows the comparison of all overlapping areas.

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