A Generalized Model of Partial Resale

Abstract

Partial resale refers to the business scenario where some resale activities, yet not all, are feasible. A paper by Geng, Wu and Whinston[12] finds that a time-restricted resale market may benefit a ticket seller more than both unrestricted resale and no resale. This paper extends the study on profitable partial resale model in two directions with both theoretical and practical importance. First, we show that the existence of arbitrage, including scalping activities by scalpers, does not reduce the benefit of partial resale to the seller. Moreover, arbitrage may increase the benefit of partial resale to the seller. Second, we propose two structures of partial resale that differ from the one with time-restricted resale market. These two new structures lead to different firm pricing strategies and consumer behaviors, yet the eventual seller profits are the same across all three alternatives. This finding implies that the seller can have flexibility in choosing how to conduct partial resale without affecting its profitability.

Keywords: price discrimination, ticket, resale, arbitrage

1. Introduction

The advancements of Internet-based marketplaces in recent years, such as StubHub and Craigslist, have enabled consumers to trade event tickets directly with each other with unprecedented ease. As a result, ticket resale has exploded in recent years (Kelsey Group [18], Happel and Jennings[15][16]). This in turn leads to great interest among marketers and academia to study *how resale affects a ticket seller's profit*.

Most of past research has focused on two polar cases regarding resale activities: one polar case is *no resale*, where resale among buyers is strictly banned; the other polar case is *complete resale*, where all tickets are exchangeable among consumers without any restriction (Deserpa[10], Williams[27], Swofford[24], Rudi, Kapur and Pyke[21], Courty[7], Karp and Perloff[17]). Geng, Wu and Whinston [12] (hereafter GWW07) argue that a third option may benefit a ticket seller more than the above two polar cases. They term this third option *partial resale*, which stipulates that *some resale activities, yet not all, are feasible*. In a stylized model consisting of two periods (*advance* and *spot*), partial resale (resale only in advance period but not spot period) may lead to a higher seller profit than both complete resale and no resale.[1]

The idea of partial resale enables marketing practitioners and policy makers to study resale markets and resale policies with a significantly widened viewpoint. Nevertheless, several additional issues need to be addressed before marketing practitioners can adopt partial resale for their business practices. One such issue is *arbitrage*. Ticket arbitrage is often associated with professional scalpers who usually do not have interest in consuming any ticket. Moreover, an ordinary consumer who intends

[1] Courty [8] also considers partial resale in a different setup, yet does not find it to be superior to no resale in terms of seller profitability.

to consume a ticket may also take part in arbitrage by purchasing multiple tickets and flipping all but one of them. GWW07 does not consider professional scalpers, and limits each consumer's purchase quantity to at most one. *Will partial resale still be valuable for the seller when arbitrage is unconstrained?* The answer to this question is important for marketing practitioners, especially when online consumer-to-consumer markets have led to explosive growth of arbitrage activities (Kelsey Group [18]).

Another issue is related to how partial resale is structured. GWW07 considers one specific structure of partial resale: resale in the advance period is feasible, while resale in the spot period is infeasible (hereafter referred to as *advance-period-only resale* for ease of exposition). In practice, nevertheless, many other structures of partial resale may exist as well. *Are there alternative structures of partial resale (besides advance-period-only resale) that benefit the seller as well?* The answer to this question will not only benefit a seller in practice in terms of choosing a structure of partial resale which fits its needs and market environment, but will also expand our understanding of how broadly the idea of partial resale applies in various market environments.

This paper studies the above two issues using a two-period game-theoretical model, which has been widely adopted in the literature on tickets and advance selling (Shugan and Xie[23], Xie and Shugan[28], Lee and Whang[19], Geng et al[12]). Different to previous literature, we introduce arbitrage in the two-period model, and consider resale markets with different restrictions.

Our analysis provides affirmative and interesting findings on these two issues. With respect to arbitrage, we show that its existence does not reduce the benefit of partial resale to the seller. Furthermore and strikingly, arbitrage can actually increase the benefit of partial resale to the seller when the number of early arrivers is less than the number of high-valuation buyers. Intuitively, partial resale enables the seller to price-discriminate high-valuation buyers regardless of their arrival times. Without arbitrage, the sale of the high-priced tickets is constrained by both the number of the early arrivers and the number of the high-valuation buyers; with arbitrage, the seller is able to price-discriminate all high-valuation buyers. The latter case thus leads to a higher profit for the seller when the number of early arrivers is less than the number of high-valuation buyers.

With respect to the structure of partial resale, we study two alternatives to advance-period-only resale. Under the first alternative, tickets sold in the advance period are resalable while the ones sold in the spot period are not. We call this alternative *advance-ticket-only resale*. Advance-ticket-only resale differs from advance-period-only resale in that the former allows the resale of advance tickets even in the spot period while the latter does not. This difference changes buyers' beliefs and their purchase and resale decisions. Consequently, the ticket seller's optimal pricing strategy under advance-ticket-only resale is significantly different from that under advance-period-only resale. Nevertheless and strikingly, we show that the seller's profits are the same under the two partial resale structures. We then consider the second alternative of partial resale, *high-price-only resale*, where the seller offers two (instead of one as before) posted prices concurrently in the advance period. Tickets with the higher price are resalable, while ones with the lower price are not. Although the seller's pricing strategy and induced consumer behavior under the high-price-only resale are distinctive to those under the aforementioned two partial resale structures, the seller profit is the same. These findings show that the seller has a much larger flexibility in choosing how to conduct partial resale without affecting its profitability.

This paper contributes to the small but growing literature on ticket. A significant number of papers in this literature (Thiel[25], Williams[27], Swofford[24], Courty[7], Karp and Perloff[17], Depken[9]) consider resale by professional scalpers, but not by ordinary consumers. These papers also assume that scalpers differ from ordinary consumers in term of market information, risk aversion, and the ability to get tickets. In contrast, our model allows both professional scalpers and ordinary consumers to buy and resell tickets. Indeed, the emergence of online consumer-to-consumer markets, such as eBay, has provided unprecedented opportunities for ordinary consumers to trade tickets as effectively as professional scalpers.

This paper also contributes broadly to the literature on price discrimination. Past research in related fields has considered pricing discrimination using purchase history (Acquisti and Varian [1]), bundling (Sankaranarayanan[22], Geng et al.[11]), advance selling (Shugan and Xie[23], Xie and Shugan[28], Cachon[4], Guo[14], Gopal et al.[13], Aron et al.[2]), and coupons and rebates (Chen et al.[5], Lu and Moorthy[20], Cheng and Dogan[6]). Resale among consumers is often deemed detrimental to seller profit and thus assumed away in prior research on price discrimination, as characterized by Tirole[26] (page 134): "if the transaction (arbitrage) costs between two consumers are low, any attempt to sell a given good to two consumers at different prices runs into the problem…" We contribute to the literature on price discrimination by showing that resale, if carefully controlled, can be an effective price discrimination tool even if the transaction costs for arbitrage is zero.

The rest of this paper is organized as follows: The model is described in the next section. We then study the impact of arbitrage on partial resale. Next, we introduce and study two alternative structures of partial resale. We then discuss the managerial implications and conclude the paper.

2. The Model

Our model consists of one monopolistic ticket seller and a large number, $\hat{\Omega}$, of buyers. The ticket seller can produce up to T tickets at a constant marginal cost normalized to zero. Tickets are sold in two periods: *advance* and *spot*. In the advance period, the ticket seller releases S_1 tickets at price p_1. The ticket seller then makes the rest of tickets available at price p_2 in the spot period.[2]

There are three types of buyers: *high-type buyers*, *low-type buyers*, and *professional scalpers*. A buyer of high-type (low-type) would like to consume at most one ticket at valuation H (L). $H > L > 0$. However, he/she may buy multiple tickets if he/she expects a non-negative profit from arbitrage on the resale market. A professional scalper is a buyer that has no intention to consume any ticket, i.e. with a valuation of zero. Let the total number of high- and low-type buyers be Ω, whereas $\Omega \leq \hat{\Omega}$. Ω is then the upper bound of market demand. Let the number of high-type buyers be $q\Omega$ (so the number of low-type buyers is $(1-q)\Omega$).

The ticket seller's spot selling is close to the consumption time. Moreover and as we discussed earlier, in practice the spot period (usually lasts days or only hours) is much

[2] If the firm cannot sell all S_1 tickets in the advance period, one issue is whether it will carry these unsold tickets to the spot period. This issue does not affect our analysis since, as we'll show later, in equilibrium where partial resale is beneficial to the firm, all S_1 tickets will be sold out in the advance period.

shorter than the advance period (usually lasts months or weeks). Therefore, it is commonly assumed in the ticket literature that, by the time the ticket seller starts spot selling, buyers know their own valuations of a ticket. In contrast, by the time the ticket seller starts advance selling, buyers (except for scalpers) do not know their own valuations. As a consequence, there is an information revelation process for each buyer from advance period to the spot period. Our model follows such an information revelation process.

In this paper we consider strategic buyers – even if a buyer can catch the ticket seller's advance selling, she may strategically opt to wait for later purchase opportunities for a better deal. Moreover, a buyer may take advantage of resale opportunities (to either sell or buy) if doing so benefits her. To model this strategic waiting behavior, we consider buyers that are differentiated by their arrival time in the market. Some buyers, called *early arrivers*, arrive early enough to catch both the ticket seller's advance and spot selling (and they can be strategic in choosing between these two purchase opportunities); the rest buyers, called *late arrivers*, arrive after the ticket seller's advance selling. Among high- and low-type buyers, let $\alpha\Omega$ of them be early arrivers, and the rest be late arrivers. For clarity, hereafter we refer to $\alpha\Omega$ as *early demand*. Due to the existence of scalpers (who may arrive early or late), early demand is no larger than the number of early arrivers. For simplicity, let a buyer's arrival time and type be independent.

Resale activities happen after the ticket seller's advance selling. Resellers can be all three types of buyers (even a high-type buyer if he/she holds more than one ticket). We assume the ticket resale price is the market clearing price determined by the aggregate supply and demand in the resale market. Because the ticket seller's spot selling changes the supply and the demand in the resale market, ticket resale prices before the spot selling may be different to that after the spot selling. Thus, we split the resale market into two over time. The resale market before the ticket seller's spot selling, if it is open, is called *advance-resale market*. That after the spot selling, if it is open, is called *spot-resale market*. This setup is also adopted by Courty [8] and GWW07.

For notational convenience, we divide the advance period and the spot period into two sub-periods: the advance period consists of *advance-selling* and *advance-resale* sub-periods; the spot period consists of *spot-selling* and *spot-resale* sub-periods. The timing of the model is as follows:

- *Advance-selling* sub-period: the ticket seller announces p_1 and releases S_1 tickets[3]; early arrivers come, and they decide whether to buy or to wait.
- *Advance-resale* sub-period: buyers learn their own valuations (except for scalpers who know their valuation – zero – from the very beginning); all rest buyers come; if the advance-resale market is open, buyers may trade tickets, and the resale price is denoted as p_{r1}.[4]
- *Spot-selling* sub-period: the ticket seller announces p_2; buyers decide whether to buy from the ticket seller.
- *Spot-resale* sub-period: if the spot-resale market is open, buyers may trade tickets before the consumption time, and the resale price is denoted as p_{r2}.

[3] Whether S_1 is observable to buyers does not matter.

[4] Similar to GWW07, since transaction-cost based arguments are not the focus of this paper, we assume that the resale market, whenever open, is perfect in that matching is costless and efficient (ref).

Note that the advance-resale sub-period can last weeks or months in practice. Therefore, all actions including learning of valuations, buyer arrival and ticket resale can happen gradually and heterogeneously for buyers. Nevertheless, in order to focus on the resale market's impact on the ticket seller's selling strategy and to simplify the analysis, in this paper we only consider the equilibrium outcome of ticket resale when all buyers' valuations are revealed.

This model extends GWW07 in two important directions.[5] First, GWW07 does not consider scalpers, and limits each high- or low-type buyer to at most one ticket. In our model considers all types of buyers (high-type buyers, low-type buyers and professional scalpers), and all of them can arbitrage tickets for profit. Specifically, a high- or low-type buyer is allowed to purchase *multiple* tickets. Since we assume that such a buyer wants to consume at most one of the tickets, the only reason for buying more than one is for arbitrage.

Second, this model extends GWW07 in considering multiple candidate structures of partial resale. GWW07 studies one structure of partial resale, *advance-period-only resale (APO hereafter)*, under which the advance-resale market is open yet the spot-resale market is closed. We consider two alternative structures of partial resale. The first is *advance-ticket-only resale (ATO hereafter)*, under which tickets sold in the advance period are always resalable while the ones sold in the spot period are not. It differs from APO in that the former allows the resale of advance tickets even in the spot period while the latter does not. The second is *high-price-only resale (HPO hereafter)*, under which the ticket seller offers all tickets at two (instead of one as before) posted prices

[5] Though the timing in our model is discreet while the one in GWW07 is continuous, they are analytically equivalent. For ease of exposition we adopt the discreet setup in this paper.

concurrently in the advance period, tickets with the higher price are resalable, and tickets with the lower price are not.

In this paper we are interested in cases where partial resale may benefit the ticket seller. There are, nevertheless, several cases under which the ticket seller's profit is always maximized under spot-only selling (i.e. $S_1 = 0$), and thus partial resale does not benefit the ticket seller: first is when $T \geq \Omega$; second is when $q \geq T/\Omega$; third is when $q \geq L/H$.[6] To rule out these uninteresting cases for succinctness of analysis, *we assume in this paper that $T < \Omega$ and $q < \min\{T/\Omega, L/H\}$*. The former assumption is called the *limited capacity* condition.

We study the impact of arbitrage on partial resale in the next section. We then analyze alternative structures of partial resale.

3. The Impact of Arbitrage on Partial Resale

In this section we study the impact of arbitrage on partial resale and the ticket seller's profitability. Allowing arbitrage has a direct impact on how many tickets early arrivers can purchase during the ticket seller's advance selling. In GWW07, if $S_1 > \alpha\Omega$, i.e. the number of advance tickets is larger than early demand, only up to $\alpha\Omega$ of these tickets can be sold. In our model, nevertheless, all S_1 tickets can be sold even if $S_1 > \alpha\Omega$ since some high- and low-type buyers may purchase multiple tickets and scalpers may jump in as well if doing so benefits them. Our key finding in this section is that this possibility of

[6] We provide detailed analysis of these three cases in the proof of Propositions 1 and 3 in the Appendix. The results are the following: if $q \geq \min\{T/\Omega, L/H\}$, the firm's profit is maximized by spot-only selling at price H; if $q < \min\{T/\Omega, L/H\}$ and $T \geq \Omega$, the firm's profit is maximized by spot-only selling at price L.

selling more advance tickets than the early demand can significantly change the ticket seller's profit under partial resale. We first show the market equilibrium under APO.

Proposition 1: *Under APO and if arbitrage exists:*

The ticket seller adopts premium advance selling with $p_2^* = L$*,* $p_1^* = L + [(1-T/\Omega)/(1-q)](H-L)$ *and* $S_1^* = q\Omega$. *All advance tickets are sold out. Moreover, the resale price in the advance resale market* $p_{r1} = p_1^*$. *The ticket seller's profit is* $TL + [q/(1-q)](\Omega - T)(H-L)$.

All proofs are in the Appendix. A key result in Proposition 1 is that the seller will release $q\Omega$ tickets during its advance selling and, more importantly, early arrivers will purchase all these tickets even though the spot price will be lower (i.e. $p_2^* < p_1^*$) and the early arrivers have the option to wait and buy later. We will first explain the intuition on why such a sold-out during advance selling can happen, as well as why partial resale plays a critical role in driving this result. We will then analyze the relationship between this sold-out result and arbitrage, and its eventual impact on the ticket seller's profit.

To understand why this sold-out at a relatively high price can happen in equilibrium, consider an early arriver's options. If an early arriver deviates from this equilibrium (i.e. choose to wait instead of to buy), she has two more opportunities to get a ticket: in the advance resale market or later during the ticket seller's spot selling. The former is no better than purchasing directly from the ticket seller during advance selling since equilibrium resale prices are the same as p_1^*. The latter is risky since rationing happens during the ticket seller's spot selling (because $T < \Omega$). In fact, the price premium, $p_1^* - p_2^*$, that the ticket seller charges during advance selling, just makes early arrivers indifferent between purchasing early or late.[7]

This result of sold-out depends on the structure of partial resale, namely, the advance resale market is open while the spot resale market is closed. Without the advance resale market, early arrivers will never purchase more tickets than the early demand – thus the ticket seller cannot sell $q\Omega$ tickets when $q > \alpha$; moreover, buyer valuation of advance tickets will decrease since they can no longer use the advance resale market to unload their holdings. If the spot resale market is open, early arrivers will strategically wait to buy later – even if there's rationing in the ticket seller's spot selling, a high-type buyer can always secure a ticket later in the spot resale market.

Arbitrage plays a critical role in driving the results in Proposition 1. Note that the amount of advance tickets sold in advance, $q\Omega$, is fixed regardless of the *early demand.* When the early demand is less than the number of advance tickets released, i.e. when $\alpha\Omega < q\Omega$, a sold-out of all advance tickets can happen only if some high- or low-type buyers purchase multiple tickets, or if scalpers participate, or both. In contrast, without arbitrage, as in GWW07, the ticket seller can sell at most $\alpha\Omega$ tickets in advance if $\alpha < q$. The next proposition shows how arbitrage affects model equilibrium and the ticket seller's profit.

Proposition 2: *Under APO and if* $\alpha < q$*:*

[7] Strictly speaking, the firm's advance price should be p_1^* minus an infinitesimal in order to ensure that early arrivers do purchase early. In this paper we adopt the common practice of omitting this infinitesimal for succinctness of discussion.

i) If arbitrage exists, all high-type buyers eventually pay the ticket seller's premium advance price of $L+[(1-T/\Omega)/(1-q)](H-L)$ *regardless of their arrival time. Rest tickets are sold at price* L.

ii) If arbitrage is limited (as in GWW07) and $q<(T/\Omega)(L/H)+(1-T/\Omega)(1-L/H)/(1/\alpha-1)$, *only* $\alpha\Omega$ *high-type buyers eventually pay the ticket seller's premium advance price of* $L+[(1-T/\Omega)/(1-\alpha)](H-L)$. *Rest tickets are sold at price* L.

iii) Otherwise, the seller adopts spot-only selling with $p_2^*=H$, *and only* $q\Omega$ *tickets are eventually sold.*

Part (i) of Proposition 2 directly follows Proposition 1 in this paper, and parts (ii) and (iii) directly follow Proposition 1 in GWW07.[8] Note that this proposition only covers the interesting case where $\alpha<q$: if $\alpha \geq q$, the ticket seller sells $q\Omega$ tickets in the advance selling[9], which is less than the number of early arrivers. Arbitrage is not necessary needed to transfer high-priced tickets to rest buyers.

Parts (i) and (ii) Proposition 2 reveal the central intuition on why partial resale (such as APO) may benefit a monopolistic ticket seller. Under partial resale the ticket seller is effectively practicing *price discrimination based on buyer valuations*: all or most high-type buyers, regardless of their arrival time, eventually pay a premium price, while

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[8] Parts (ii) and (iii) can also be derived from the proof of Proposition 1 in this paper's Appendix: if we change the constraint on S_1^* from $S_1^* \leq q\Omega$ to $S_1^* \leq \min\{q\Omega, \alpha\Omega\}$, these results will follow.

[9] The ticket seller cannot sell more than $q\Omega$ tickets in the advance selling. Otherwise, the ticket resale price will fall to L, and no buyer would like to buy from the advance selling when anticipating $p_{r1}=L$.

all low types either pay a price of L or end up not having a ticket.[10] In contrast, without partial resale, the best the ticket seller can do is to price discriminate buyers base on arrival times. Partial resale can lead to a better firm profit than no resale if price discrimination based on buyer valuation is more effective than price discrimination based on buyer arrival times.

Proposition 2 also reveals the critical impact of arbitrage on the effectiveness of price discrimination. A comparison of parts (i) and (ii) in Proposition 2 leads to two observations. First, the ticket seller is able to price discriminate *more* high-type buyers when arbitrage exists. Intuitively, anticipating that more high-type buyers will arrive later and will be willing to purchase tickets in the advance resale market, early arrivers have incentive to purchase tickets for the purpose of arbitrage, which the ticket seller can then take advantage of by releasing more tickets than the early demand. Second, the ticket seller is able to charge a *higher* advance price when arbitrage exists. Both factors imply that arbitrage by consumers can increase the effectiveness of price discrimination, compared to the case without scalping. This result is summarized in the following corollary.[11]

Corollary 1: *Given APO, when* $\alpha < q$*, the existence of arbitrage increases the ticket seller's profit.*

[10] We use the word "eventually" to emphasize that not all high-type buyers purchase tickets at the same time: some purchase during the firm's advance selling; other purchase in the advance resale market.
[11] The proof of Corollary 1 also requires a comparison of seller profits under parts (i) and (iii) of Proposition 2. The result is straightforward given that $q < T / \Omega$ and $q < L / H$.

Corollary 1 is a surprising result since arbitrage (especially if done by professional scalpers) is commonly viewed as detrimental to the ticket seller's profit. This result shows that, contrary to the common belief, arbitrage can benefit a ticket seller by enabling it to more effectively price discriminate buyers. Specifically, the ticket seller is able to pocket all profit gains from discriminating all high-type buyers even before some of them (i.e. high-type late arrivers) arrive in the market. Scalpers (and also low- and high-type buyers who purchase multiple tickets) acts as a media during this price discrimination process: they first pay the ticket seller the premium price on advance tickets, they then recoup the loss by selling later to high-type buyers at the same price.[12]

<Insert Figure 1 about here>

To illustrate the results in Proposition 1, Proposition 2 and Corollary 1, we present a numerical example where $\Omega = 1$, $T = 0.8$, $H = 2$ and $L = 1$. We consider a domain[13] where q varies between 0 and 0.5, and α varies between 0 and 1. We calculate the seller's profit under different resale structures in this domain. As shown in Figure 1, when the seller adopts partial resale, it gains more profit than that under complete resale. When $\alpha < q$ (the space *0ABC* in Figure 1-2, 1-4), the seller's profit increases if arbitrage is allowed in the resale market.

We next turn our attention to two alternative structures of partial resale other than APO. Hereafter we assume that scalping always exists, and as a result it is possible for the ticket seller to sell more advance tickets than the early demand.

[12] Note that we assume a perfect resale market in which resale is costless. In practice, however, resale can be costly and scalpers would expect a non-zero profit. To deal with these issues, the firm can slightly lower its advance price so that the spread between the advance price and the resale price provides enough incentive for resale activities. As long as this spread is small, our results will not qualitatively change.

[13] Recall in this paper we assume $q < \min\{T/\Omega, L/H\}$

4. Alternative Structures of Partial Resale

Until now we only discussed one peculiar structure of partial resale – *advance-period-only resale (APO)*. APO helps illustrate the value of partial resale to a ticket seller, yet it is not the only available structure of partial resale. In this section we consider two alternative structures of partial resale. The first is *advance-ticket-only resale (ATO)*, under which tickets sold in the advance period are always resalable while the ones sold in the spot period are not. The second is *high-price-only resale (HPO)*, under which the ticket seller offers all tickets at two (instead of one as before) posted prices concurrently in the advance period, tickets with the higher price are resalable, and tickets with the lower price are not. Analyzing these two alternatives help us answer two questions: *Are there alternative structures of partial resale (other than APO) that also benefit the ticket seller? And if so, which structure of partial resale benefits the ticket seller the most?* This section answers these two questions.

We first characterize the market equilibrium under ATO, as shown below, and compare it to the one under APO. We then discuss HPO.

Proposition 3: *Under ATO and if arbitrage exists:*

The ticket seller adopts premium advance selling with $p_2^* = L$, $p_1^* = H$ *and* $S_1^* = [q/(1-q)](\Omega - T)$. *All advance tickets are sold out. Moreover, ticket resale price in the advance resale market and that in the spot resale markets are the same price as* p_1^*.

The ticket seller's profit is $TL + [q/(1-q)](\Omega - T)(H - L)$.

A comparison of Propositions 3 and 1 reveals several important results regarding how different structures of partial resale can lead to different firm and consumer behaviors. First, ATO and APO induce the ticket seller to adopt different pricing strategies: the advance price under ATO (H) is higher than that under APO ($L+[(1-T/\Omega)/(1-q)](H-L)$). In other words, the ticket seller is able to command a higher per-ticket profit margin in the advance period under ATO than under APO. To understand this, note that only ATO allows an advance ticket to be resold in the spot period. Under ATO, the equilibrium spot resale price is H if there are fewer resellers than high-type buyers without a ticket, and L otherwise. The ticket seller prefers the former choice since otherwise, anticipating the chance to buy a ticket at price L in the spot resale market, no buyer will pay the ticket seller a price higher than L at any time earlier. A spot resale price of H leads to an advance resale price of H as well since a reseller can always wait to sell later. Anticipating this, the ticket seller then charges an advance price of H so it extracts all surpluses from resellers. In contrast, under APO there is no spot resale market and thus a reseller does not have the luxury to wait and sell in the spot period. As a result, the reseller has to settle at a resale price lower than H during the advance resale sub-period. This in turn implies that the ticket seller has to charge an advance price less than H in order to motivate potential resellers to buy in the first place.

Though ATO leads to a higher per-ticket profit margin in the advance period than APO, it may not (and does not as we discuss shortly) translate into a higher overall profit. The second result following Propositions 3 and 1 is that the ticket seller sells fewer advance tickets under ATO than under APO. This again is because of the possibility of spot resale under ATO: the ticket seller has to limit the number of advance tickets it sells in order to induce a high resale price in the spot period. In contrast, under APO the ticket seller does not face such a constraint.

The third and surprising result is that, though ATO and APO lead to quite different ticket seller and reseller behaviors as just discussed, they eventually benefit the ticket seller equally. Intuitively, ATO (as compared to APO) enables the ticket seller to charge a higher advance price, yet at the same time the ticket seller can sell only fewer advance tickets. This result says that these two factors cancel each other out in equilibrium, and as a result ATO leads to the same ticket seller's profit as APO.

Next we discuss another alternative structure of partial resale, HPO. Recall that under HPO the ticket seller offers all tickets at two (instead of one as before) posted prices concurrently in the advance period, tickets with the higher price are resalable in both advance and spot periods, and tickets with the lower price are not. It seems HPO and ATO are significantly different because of how many posted prices the ticket seller can charge in a single period (advance): two concurrent ones under HPO, and only one under ATO. Nevertheless, we show in the next proposition that the market equilibrium is quite similar under HPO and ATO, and as a result HPO leads to a ticket seller's profit same as the one under ATO or APO.

Proposition 4: *Under HPO, if arbitrage exists and if the ticket seller releases all tickets in the advance period:*

The ticket seller releases $[q/(1-q)](\Omega - T)$ *resalable tickets at price* H*, and the rest tickets as non-resalable tickets at price* L*. Moreover, ticket resale prices in the* *advance resale market and spot resale market are* H *. The ticket seller's profit is* $TL + [q/(1-q)](\Omega - T)(H - L)$.

A comparison of Propositions 4 and 3 shows that, on the ticket seller's side, the pricing and quantity decisions are quite similar: under both HPO and ATO, the ticket seller would release $[q/(1-q)](\Omega - T)$ tickets at price H during its advance selling. Furthermore, resale prices are all H under both HPO and ATO. The only difference is when the ticket seller releases low price tickets: under HPO low price tickets are released at the beginning of the advance period, while under ATO low price tickets are released at the beginning of the spot period. This difference, nevertheless, has little impact on resale activities since the low price tickets are *not* resalable under either HPO or ATO.[14] As a result, HPO and ATO lead to similar market dynamics and eventually the same ticket seller's profit. We highlight this last result (as well as the result that the ticket seller's profits are the same under ATO and APO) in the following proposition.

Proposition 5: If *arbitrage exists, all three alternatives of partial resale (APO, APO and HPO) lead to the same ticket seller's profit.*[15]

Proposition 5 follows directly from a comparison of Propositions 1, 3 and 4. This is a strong and surprising result with several implications. First, APO, as proposed in GWW07, is not the only structure of partial resale that can benefit the ticket seller. As a

[14] Under ATO, since we know from Proposition 3 that spot tickets are priced at L, "spot tickets are not resalable" is equivalent to "low price tickets are not resalable."
[15] Straightforward calculation shows that all three alternatives of particle resale lead to the same consumer surplus as well.

result, the ticket seller can implement partial resale in a more flexible way than previously known. Such flexibility can be important for the ticket seller since not all structures of partial resale are equally easy to implement – we will expand on this point next in the Managerial Implications section.

Second and surprisingly, the ticket seller not only can be flexible in choosing how to implement partial resale, it can do so without worrying about which one leads to a better profit – as far as APO, ATO and HPO are concerned, they all lead to the same ticket seller's profit despite different equilibrium firm and consumer behaviors.

The result that all three structures of partial resale can benefit the ticket seller can be intuitively explained by the commonality among all three structures of partial resale: low price tickets cannot be resold under either structure. As we mentioned at the very beginning of this paper, ticket resale may affect the ticket seller's profit in both ways: on the up side, a consumer is willing to pay more for a resalable ticket than a non-resalable one; on the down side, the existence of resalable tickets may cut into a seller's total sales since some consumers will buy from a reseller instead of the seller. Banning resale of low price tickets helps the ticket seller control the aforementioned down side risk since resale of high price tickets will not cut into the ticket seller's sales of low price tickets (while the other way around is not true).

5. Managerial Implications

In this section we discuss the managerial implications of our research – first about arbitrage, and then about alternative structures of partial resale.

Arbitrage

We found out that arbitrage does not lead to decreased the ticket seller's profit under partial resale. Rather, it increases the ticket seller's profit in some cases. On the surface this result looks surprising since the event ticket industry in general has a negative view of arbitrage (especially if done by scalpers). However, a close examination of how arbitrage affects buyer behavior reveals that our result is actually quite intuitive. First of all, note that under partial resale not all arbitrage activities are feasible – *flipping of low price tickets is not allowed under either APO, ATO, or HPO*. Second, allowing scalping in the advance period enables the ticket seller to *sell more advance tickets than the total early demand* – the extra tickets are bought by scalpers or other buyers who feel that they can recoup their cost later by selling the same tickets to late high-type arrivers. As a result, the ticket seller is effectively price discriminating late high-type buyers even before they arrive in the market. During this process, resellers are effectively utilized as a price discrimination tool by the ticket seller.

This result favoring arbitrage from a business standpoint is consistent with recent developments regarding ticket resale. In the last several years many states in the United States have abolished or loosened their anti-scalping laws, or are in the process of doing so.[16] At the same time, many businesses, such as the New England Patriots of the National Football League, are changing their stance from resisting ticket resale to actively facilitating and participating in ticket resale.

When arbitrage exists, our research offers several suggestions for businesses. First, a ticket seller should carefully control what can and cannot be scalped. Scalping of low

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[16] In 2007 alone, five states including Connecticut, New York, Minnesota, Pennsylvania and Missouri repealed their anti-scalping laws (Branch [3]).

price tickets cuts into the ticket seller's profit and should be prevented. Instead, the ticket seller should allow scalping of high price tickets only. Second, when early demand is low (i.e. most buyers arrive in the market long after the ticket seller's advance selling) and as long as the advance price is higher than the spot price, the ticket seller can consider releasing more advance tickets than the early demand with the expectation that the extra supply will be absorbed by scalpers (and the ticket seller can profit from this).

Alternative Structures of Partial Resale

We found that different structures of partial resale lead to significantly different firm and buyer behaviors in equilibrium. Under ATO and HPO (as compared to APO), the ticket seller should charge a higher advance price and at the same time release fewer advance tickets. Intuitively, the higher advance price under ATO and HPO is due to the fact that a premium price ticket can be resold in a wider timeframe (both advance and spot) under ATO and HPO, compared to advance period only under APO. Buyers observe a higher resale price under ATO and HPO than the one under APO. For all three structures, a key idea for the ticket seller is to release a limited number of advance tickets so that supply is smaller than demand in the resale market, and thus a relatively high resale price can be sustained to stimulate early purchase.

Despite different resulting firm and buyer behaviors, all three structures of partial resale eventually benefit the ticket seller equally. This strong result immediately implies that, whenever APO benefits the ticket seller (as discussed in detail in GWW07), ATO and HPO can also benefit the ticket seller. This result also significantly expands our understanding regarding how widely applicable the idea of partial resale is – it is not limited to any specific structure of implementation.

Having the flexibility to pick a structure of partial resale that fits its business needs and environment is often critical for marketing practitioners since not all structures are equally easy to implement. The two structures proposed in this paper, ATO and HPO, can be implemented in a rather straightforward way by a ticket seller. For ATO, the ticket seller can require the name of the customer be printed on a spot ticket (yet not required for an advance ticket), and then require the printed name on the ticket to match that on a customer's ID. For HPO and similarly, the ticket seller can required printed names on all low price tickets. Given technological advancements in recent years, these measures are not hard to implement either online or at a physical ticket booth: nowadays many concert and professional sport organizers sell tickets with consumer names attached.

The structure proposed in GWW07, APO, is relatively harder to enforce in the aforementioned way since the resalability of a ticket is supposed to change at a certain time (the end of the advance period). In practice, however, it is hard for the ticket seller to tell when a ticket is transferred from one consumer to another one. APO, nevertheless, has its own advantage over ATO and HPO in some cases. One such case, as described in GWW07, is when advance and spot resale takes place in different channels, such as advance resale online and spot resale offline. The ticket seller can then allow online resale but ban physical resale to implement APO. This implementation does not require the ticket seller to print names on tickets or to verify IDs during admission. To summarize, ATO, HPO and APO each has its own advantages. Our research shows that the ticket seller can have the flexibility to pick the structure that fits its needs without worrying how they affect profits differently.

6. Concluding Remarks

Our research extends the theory of partial resale by considering arbitrage and by considering alternative structures to implement partial resale. Our results show that arbitrage does not reduce the ticket seller's profit under partial resale. Rather, arbitrage increases the ticket seller's profit in some cases. We also show that all three alternative structures of partial resale can benefit the ticket seller. As a result, the ticket seller has the flexibility to choose among these alternatives as it sees fit without affecting the effectiveness of partial resale. Our findings not only are practically important for marketing practitioners and policy makers that are concerned about ticket resale, they also contribute to the theory of partial resale by significantly expanding the applicability of partial resale – partial resale can benefit a ticket seller in a broader range of cases than previously known.

Appendix

Proof of Proposition 1:

(Part 1 of Proof of Proposition 1)

We prove an extended version of Proposition 1 as shown below, where we do NOT restrict that $T < \Omega$ and $q < \min\{T/\Omega, L/H\}$. We then show that, if $T \geq \Omega$, or $q \geq T/\Omega$, or $q \geq L/H$, the ticket seller's profit is maximized by spot-only selling – and thus partial resale won't benefit the ticket seller in these cases.

First note that, if $q \geq T/\Omega$, the ticket seller can maximize its profit simply by charging H and sell all tickets to high-type buyers on spot. Thus hereafter we can focus on the case where $q < T/\Omega$. Second note that, since there is no demand for any capacity beyond Ω, the case of $T \geq \Omega$ can be replaced, without loss of generality, with the case of $T = \Omega$. We call $T = \Omega$ abundant capacity, and $T < \Omega$ limited capacity.

(Extended Version Of) Proposition 1: *Under APO and if scalping exists:*

i) *If $q \geq L/H$, the ticket seller adopts spot-only selling with $p_2^* = H$. The ticket seller's profit is $q\Omega H$.*

ii) *If $q < L/H$ and the ticket seller has an abundant capacity, i.e. $T \geq \Omega$, it adopts spot-only selling with $p_2^* = L$. The ticket seller's profit is TL.*

iii) *If $q < L/H$ and the ticket seller has a limited capacity, i.e. $T < \Omega$, it adopts premium-advance selling with $p_2^* = L$, $p_1^* = L + \dfrac{1 - T/\Omega}{1-q}(H - L)$ and $S_1^* = q\Omega$.*

Moreover, all resellers in advance resale charge the same price as p_1^**. The ticket seller's profit is* $TL + \frac{q}{1-q}(\Omega - T)(H - L)$.

(Part 2 of Proof of Proposition 1)

The rest of the proof proceeds in two parts. In this part and as a preparation, we first prove Lemmas 1-5 in a backward order with respect to the timeline. We then prove Proposition 1 in the next part. Let π_1 denote the ticket seller's profit from advance selling and π_2 its profit from spot selling. Denote M_1 as the number of tickets traded in advance resale.

Lemma 1. *In the spot period: if* $\frac{L}{H} \leq \frac{q\Omega - qS_1 - M_1}{T - S_1}$*, then* $p_2 = H$ *; otherwise* $p_2 = L$.

Proof: In the spot period all buyers know their own valuations. The seller's spot selling is the last chance for buyers to purchase a ticket since spot resale is infeasible. Consequently, the seller will set its optimal price, p_2^*, at either *H* or *L*, depending on whether it wants to serve only high-valuation buyers or all buyers. If $p_2 = H$, $\pi_2 = (q\Omega - qS_1 - M_1)H$. If $p_2 = L$, then $\pi_2 = (T - S_1)L$. This lemma follows a comparison of these two numbers.

Lemma 2. *The seller cannot do better by equal-price advance selling than by spot-only selling.*

Proof: Suppose in equilibrium the seller sets $p_1 = p_2$, and $S_1 > 0$. If $p_1 = p_2 = H$, no one buys at the seller's advance selling since $qH + (1-q)L < p_1$ and resale price can never be higher than H. Thus the seller is effectively practicing spot-only selling.

If $p_1 = p_2 = L$, all buyers will buy whenever possible. The seller's profit is TL, which is the same it gets under spot-only selling with price L.

Lemma 3. *All resale prices in advance resale are the same (denoted as p_{r1}).*

Proof: If there are more low-valuation ticket resellers in advance resale than high-valuation buyers without a ticket, competition on the resellers' side will drive resale price down to the time-invariant price L.

Otherwise, denote the negotiated price of the *n*'th ticket resold in advance resale as $P_{r1}(n)$, where $1 \le n \le M_1$. For convenience, we call the reseller of the *n*'th ticket the *n*'th reseller. Now we want to show that $P_{r1}(n) = P_{r1}(M_1)$ holds true for any $n < M_1$. $P_{r1}(M_1)$ is the price of the last ticket resold in advance resale, and $P_{r1}(M_1)$ is determined such that a high-valuation buyer is indifferent about buying from the M_1 ' th reseller or waiting for the monopolistic seller's spot selling.

(i) If for some $n < M_1$ we have $P_{r1}(n) < P_{r1}(M_1)$, then the best strategy for the *n*'th reseller is to refuse to sell until the price is high because there are always high-valuation buyers without a ticket.

(ii) If for some $n < M_1$ we have $P_{r1}(n) > P_{r1}(M_1)$, then a high-valuation buyer without a ticket should refuse to buy since she will be better off waiting for the spot selling.

Therefore, $P_{r1}(n) = P_{r1}(M_1)$ holds true for any $n < M_1$, thus we can denote all resale prices in advance resale using a time-invariant variable, p_{r1}.

Lemma 4. *If the seller adopts advance selling (either premium or discount), we have* $S_1 \le q\Omega$ *and* $M_1 = (1-q)S_1$. *Moreover, if* $\frac{L}{H} \le \frac{q\Omega - S_1}{T - S_1}$, $p_{r1} = H$. *Otherwise* $p_{r1} = L + \frac{\Omega - T}{\Omega - S_1}(H - L)$.

Proof: We first show that $S_1 \le q\Omega$. Assume, on the contrary, that the seller adopts advance selling and sets $S_1 > q\Omega$ in equilibrium. Then $(1-q)S_1 > q\Omega - qS_1$, i.e. in advance resale there are more low-valuation ticket holders than high-valuation buyers who have no tickets. As the resellers are assumed as price-takers, the equilibrium price in advance resale will be driven down to *L* because of competition among resellers. Since buyers expect enough tickets in the resale market at price *L*, they will refuse to pay more than *L* at the seller's advance selling. As a result, $p_1 = p_2 = L$, which contradicts the assumption that the seller adopts (premium or discount) advance selling.

Since $(1-q)S_1 \le q\Omega - qS_1$, a reseller will always be able to find a buyer if it offers a low enough price. Because advance resale provides a reseller with the only chance to resell, she has the incentive to do so, thus $M_1 = (1-q)S_1$.

p_{r1} should make a buyer indifferent between buying from a reseller or from the seller's spot selling. When $\frac{L}{H} \le \frac{q\Omega - S_1}{T - S_1}$, $p_2 = H$ (recall Lemma 1), thus $p_{r1} = H$. When $\frac{L}{H} > \frac{q\Omega - S_1}{T - S_1}$, $p_2 = L$. Since all buyers will try to secure a ticket in the spot market, a high-valuation buyer only has $\frac{T - S_1}{\Omega - S_1}$ chance to get one. From $H - p_{r1} \geq \frac{T - S_1}{\Omega - S_1}(H - L)$,

we have $p_{r1} = L + \frac{\Omega - T}{\Omega - S_1}(H - L)$.

Lemma 5. *If the seller adopts advance selling, its optimal advance price is* $p_1 = p_{r1}$.

Proof: An early arriver's expected surplus from advance buying is *ESA* = $q(H - p_1) + (1-q)(p_{r1} - p_1)$, and her expected surplus from waiting is *ESW* = $q(H - p_{r1})$. To induce early arrivers to buy, the seller must pick p_1 such that *ESA* $\geq$ *ESW*. Therefore, to maximize its profit, the seller will pick p_1 such that *ESA* = *ESW*.

We are now ready to prove Proposition 1.

(Part 3 of Proof of Proposition 1)

From Lemmas 1, 4 and 5 we have three alternatives:

a) If $\frac{L}{H} \leq \frac{q\Omega - S_1}{T - S_1}$ holds, $p_1 = p_{r1} = p_2 = H$, and the seller's total profit is $\pi = q\Omega H$. In this case the seller can set $S_1 = 0$ and simply spot-sell at price H.

b) If $\frac{L}{H} > \frac{q\Omega - S_1}{T - S_1}$ holds and $T = \Omega$, $p_1 = p_{r1} = p_2 = L$, and the seller's total profit is $\pi = TL$. In this case the seller can set $S_1 = 0$ and simply spot-sell at price L.

c) If $\frac{L}{H} > \frac{q\Omega - S_1}{T - S_1}$ holds and $T < \Omega$, $p_2 = L$, $p_1 = p_{r1} = L + \frac{\Omega - T}{\Omega - S_1}(H - L)$, and the seller's total profit is $\pi = TL + \frac{S_1}{\Omega - S_1}(\Omega - T)(H - L)$, which is an increasing function of S_1.

When $T \geq \Omega$ (abundant capacity), the seller picks the alternative among (a) and (b) that gives it the highest profit. This leads to case (i) or (ii) in the proposition. Now consider $T < \Omega$, where the seller picks between alternatives (a) and (c). Since in the alternative (c), the seller's total profit is an increasing function of S_1, and from Lemma 4 we have $S_1 \leq q\Omega$, the seller sells $S_1 = q\Omega$ tickets in the advance selling.

When $q < \frac{L}{H}$ holds, $TL + \frac{q}{1-q}(\Omega - T)(H-L) > q\Omega H$, the seller will prefer alternative (c) over (a). This is case (iii) in the proposition.

When $q \geq \frac{L}{H}$ holds, $TL + \frac{q}{1-q}(\Omega - T)(H-L) \leq q\Omega H$, Therefore the seller prefers alternative (a) over (c). This is case (i) in the proposition.

Proof of Proposition 3:

For rigor, similar to the proof of extended version of Proposition 1, here we do NOT restrict that $T < \Omega$ and $q < \min\{T/\Omega, L/H\}$. This proof goes in two parts. In part 1 we prove a few Lemmas. In part 2 we prove this proposition.

(Part 1 of Proof of Proposition 3)

Let π_1 denote the seller's profit from advance selling and π_2 its profit from spot selling. Denote S_1, S_{r1}, S_2 and S_{r2} as the supplies in the advance selling, advance resale market, spot selling and spot resale market. $S_1 + S_2 \leq T$, and $S_1 \geq S_{r1} + S_{r2}$. Denote D_1, D_{r1}, D_2 and D_{r2} as the demands from high-valuation buyers in the advance selling, advance resale market, spot selling and spot resale market.

Lemma 6: *At the sport resale market, the resale price* p_{r2} equals H if $D_{r2} \geq S_{r2}$, and equals to L if $D_{r2} < S_{r2}$.

Proof: The scalpers have no interest to buy the tickets in the spot resale market. The potential buyers have valuations either as H or L. if $H > p_{r2} > L$, only buyers with valuation H would like to buy the tickets. A reseller can resell the same amount of tickets at H as at p_{r2}. Thus, $H > p_{r2} > L$ could not be an equilibrium market price. p_{r2} equals H if $D_{r2} \geq S_{r2}$, and equals to L if $D_{r2} < S_{r2}$.

Lemma 7: *At the spot selling, the spot price* p_2 *equals to H or L.*

Proof: At spot selling, as tickets are not resalable, scalpers have no interest to buy tickets. The low-valuation buyers and high-valuation buyers buy at most one ticket each.

If expecting $p_{r2} = L$, the seller sets $p_2 = L$. Otherwise low-valuation buyers will not buy tickets. Neither do the high-valuation buyers as they can wait and get tickets at a lower price in the spot resale market[17].

If expecting $p_{r2} = H$, the seller sets p_2 either as H or L. If $p_2 = L$, all high-valuation buyers and low-valuation buyers would like to buy tickets. Rationing happens. As the tickets sold at the spot selling are not resalable, each buyer demands only one ticket for her own consumption. Thus, the probability of a buyer buying a ticket is $\frac{S_2}{\Omega}$.

[17] By assuming that resale market is perfect in the sense that no Pareto improvement is possible any other way, high-valuation buyers have a priority to low-valuation buyers to get tickets if total supply is less than the total demand in the resale market. Therefore, when spot resale price is low, high-valuation buyer will always get tickets.

When the seller sells at spot price $H \geq p_2 > L$, then the low-valuation buyers will not buy the tickets. All high-valuation buyers would like to buy the tickets. Thus, the seller always has an incentive to set $p_2 = H$

Lemma 8: *The advance resale price equals to the spot resale price,* $p_{r1} = p_{r2}$.

Proof: As only advance tickets are resalable, all resellers are price-takers and in both resale markets all buyers share the same information on demand and supply, the market price in the two resale markets must be same. Otherwise, if $p_{r1} > p_{r2}$, buyers expecting a lower spot resale price, will strategically wait. Demand in the advance resale market drops and advance resale price decreases. If $p_{r1} < p_{r2}$, advance-ticket holders will wait to sell in the spot resale market. The supply in the advance resale market shrinks and advance resale price increases.

Lemma 9: *In the advance selling, the advance price converges to the spot resale price.* $p_1 = p_{r2}$.

Proof: If $p_{r2} = H$, $D_{r2} > S_{r2}$. An advance-ticket holder can always resell her holdings at *H* in the spot resale market. Then the seller can always set $p_1 = H$, Strictly speaking, it should be *H* minus a penny, without lose the demand. If $p_{r2} = L$, $D_{r2} < S_{r2}$. There are sufficient supply in the spot resale market at the price *L*. The highest advance price the seller can charge is *L*. Otherwise, the early buyers would rather wait to buy at the spot resale market.

Lemma 10: *Who holds the advance-tickets does not matter the equilibrium outcome in spot resale market.*

Proof: From Lemma 7, the spot resale price can only be *H* or *L*. Then the question is whether the demographic composition of advance-ticket holders affects the relationship of D_{r2} and S_{r2}.

Apparently all scalpers and low-valuation advance-ticket holders will sell their holdings of advance-tickets. A high-valuation advance-ticket holder may hold $n \geq 1$ tickets in the spot resale market, but at most one non-resalable ticket. If she holds one non-resalable ticket, she will consume the ticket and reseller all her holdings of advance-ticket. If all her holding are advance-tickets, she may resell *n* or *n-1* tickets. However her decision will not change the spot resale price as both D_{r2} and S_{r2} will increase or decease by one unit. Therefore, although some advance-tickets are kept by high-valuation holders but not resold in the spot resale market, the market resale price equals to the spot resale price when all advance-tickets are resold. The demographic composition of advance-ticket holders does not affect the spot resale price.

(Part 2 of Proof of Proposition 3)

As the seller try to maximize its profit, from Lemmas 6, 7, 8 and 9, its selling strategy can lead to three possible equilibrium outcomes[18].

a) $p_1 = p_{r1} = p_2 = p_{r2} = L$.

b) $p_1 = p_{r1} = p_2 = p_{r2} = H$

c) $p_1 = p_{r1} = p_{r2} = H$ and $p_2 = L$.

[18] More precisely, there can be more equilibria as the resellers can trade with each other without affecting market price and their profits. In the paper, we define an equilibrium outcome as a set of equilibrium advance price, advance resale price, spot price and spot resale price.

In outcome (a), the seller will sell all its capacity out. $S_1 + S_2 = T$. Its profit is $\pi^a = T \cdot L$. To realize this outcome, the seller can simply sell all capacity in the advance selling at *L*.

In outcome (b), as $p_{r2} = H$, $S_{r2} \le D_{r2}$. Given Lemma 10, we consider a case where all advance tickets are resold at the spot resale market, or $S_{r2} = S_1$. From Lemma 7, all high-valuation buyers would like to buy in the spot selling and no low-valuation buyer or scalper buy in the spot selling. Thus S_2 high-valuation buyers hold non-resalable tickets in the spot resale market, or $D_{r2} = q\Omega - S_2$. Therefore, $q\Omega - S_2 \ge S_1$, or $S_1 + S_2 \le q\Omega$. The seller at most sells $q\Omega$ tickets totally. (The seller can sell all her capacity at *H* in the spot selling, however, only $q\Omega - S_1$ tickets are sold out, the marginal high-valuation buyer will strategically wait to buy at the spot resale market). Its maximum profit under this equilibrium outcome is $\pi^b = q\Omega \cdot H$.

Comparing the outcome (a) and (b), the seller prefer outcome (b) if and only if $\pi^b \ge \pi^a$. As $q \le \frac{T}{\Omega}$, the necessary and sufficient condition is $q \ge \frac{L}{H}$

In outcome (c), as $p_2 = L$, the seller sells all its remaining capacity in order to maximize its profit, $S_2 = T - S_1$. Then the seller's profit is $\pi^c = S_1 \cdot H + S_2 \cdot L = TL + S_1\left(H - L\right)$. Apparently, the seller would like increase S_1 as much as possible. However, too many advance tickets imply large supply in the spot resale market, and cause the spot resale price collapse to *L*. Therefore S_1 must satisfy the following constraint.

As $p_{r2} = H$, $S_{r2} \le D_{r2}$. Given Lemma 10, we consider a case where all advance tickets are resold at the spot resale market, or $S_{r2} = S_1$. At the $p_2 = L$, all low-valuation buyers and high-valuation buyers would like to buy at the spot price *L*. The probability for a buyer to secure a ticket is $\frac{S_2}{\Omega}$. Therefore, in the spot resale market,

$D_{r2} = q\Omega\left(1 - \frac{S_2}{\Omega}\right) = q\Omega - qS_2$. Thus, $q\Omega - qS_2 \ge S_1$. Recall $S_2 = T - S_1$,

$q\Omega - q(T - S_1) \ge S_1$, or $S_1 \le \frac{q}{1-q}(\Omega - T)$. Thus, the seller's maximum profit in outcome (c) is $\pi^c = TL + \frac{q}{1-q}(\Omega - T)(H - L)$, when $S_1 = \frac{q}{1-q}(\Omega - T)$.

Comparing the outcomes (b) and (c), the seller set $p_2 = L$ instead of $p_2 = H$ if and only if that the seller's spot selling profit at $p_2 = L$ is higher than that at $p_2 = H$, or $L \cdot (T - S_1) \ge H \cdot (q\Omega - S_1)$. Substitute $S_1 = \frac{q}{1-q}(\Omega - T)$ in it. The necessary and sufficient condition of $\pi^c > \pi^b$ is $q\Omega H < TL + \frac{q}{1-q}(\Omega - T)(H - L)$, or $q < \frac{L}{H}$.

Comparing outcomes (a) and (c), $\pi^c \ge \pi^a$. When $T \to \Omega$, $\pi^c \to \pi^a$.

Therefore, we have the following conclusion:

Under partial resale (Advance-ticket-only) with professional scalpers:

i) If $q \ge \frac{L}{H}$*, the seller sells all tickets at H,* $p_1^* = p_2^* = H$*. Seller profit is* $q\Omega H$.

ii) If $q < \frac{L}{H}$ *and the seller has an abundant capacity, i.e.* $T = \Omega$*, it sells all tickets at L,* $p_1^* = p_2^* = L$*. Seller profit is* TL.

iii) If $q < \frac{L}{H}$ *and the seller has a limited capacity, i.e.* $T < \Omega$*, it adopts premium-advance selling with* $p_1^* = H$ *,* $S_1^* = \frac{q}{1-q}(\Omega - T)$ *and* $p_2^* = L$ *,. Seller profit is*

$TL + \frac{q}{1-q}(\Omega - T)(H - L)$.

Proof of Proposition 4:

Under HPO, the seller releases S_h tickets at a high price p_h and S_l tickets at a low price p_l, where $L \le p_l < p_h \le H$ and $S_h + S_l \le T$. High-priced tickets are resalable and low-priced tickets are not.

First of all, as the seller's profit is $\pi = S_h \cdot p_h + S_l \cdot p_l$ and $T \le \Omega$, in order to maximize its profit, the seller would like to sell as many tickets as possible. That is, $S_h + S_l = T$.

Second, we have $p_h = H$ and $p_l = L$ because of the following reason. As $p_h > L$, only high-valuation consumers are the final buyers of high-priced tickets. Therefore, the resale price of the high-priced tickets must be *H* as the resellers pursue the highest profits. Thus, the seller will set $p_h = H - \varepsilon$ to maximize its profit. Meanwhile, if $p_l > L$, only high-valuation consumers would like to buy low-priced tickets, then the seller only sells tickets to high-valuation consumers. Its profit can not be more than that if it simply sells tickets at *H.* Therefore, $p_l = L$.

Next we derive the seller's best HPO strategy. As $p_h = H$ and $p_l = L$, high-valuation consumers would like first try to buy low-priced tickets. Because low-priced tickets are not resalable, each consumer demands one ticket and scalpers do not. As we assume that in both advance and spot period q of the new consumers are high-valuation consumers, then no matter whether the low-priced tickets are sold out or not in the advance period, a high-valuation consumer and a low-valuation consumer have the same chance to get a low-priced ticket. Therefore, among the S_l tickets sold at low price, qS_l tickets are purchased by the high-valuation consumers.

Because high-priced tickets are resalable, S_h high-priced tickets are eventually sold to rest $q\Omega - qS_l$ high-valuation consumers. Then, in order to maximize its profit, the seller should set $S_h = q\Omega - qS_l$. As $S_h + S_l = T$, $S_h = \frac{q}{1-q}(\Omega - T)$ and $S_l = \frac{T - q\Omega}{1-q}$.

And the seller's profit is $\pi = S_h \cdot H + S_l \cdot L = TL + [q/(1-q)](\Omega - T)(H - L)$.

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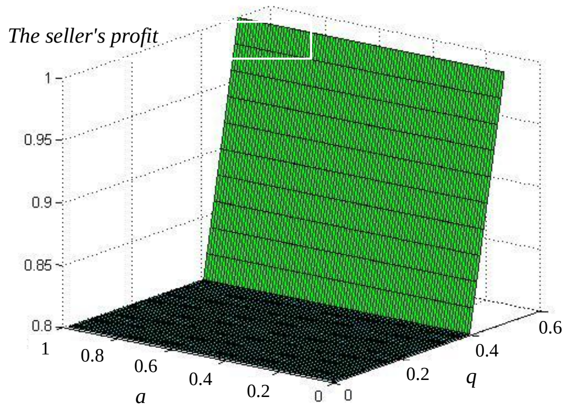
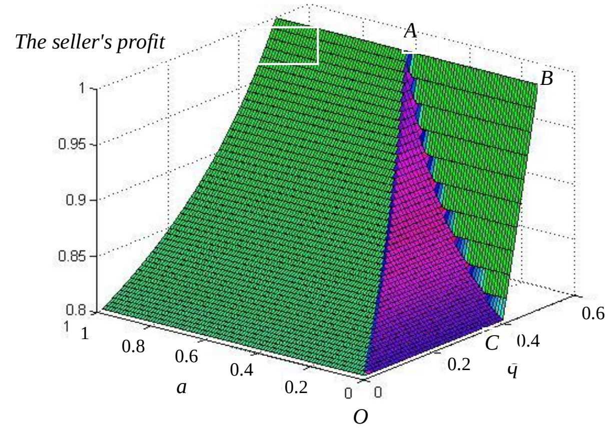
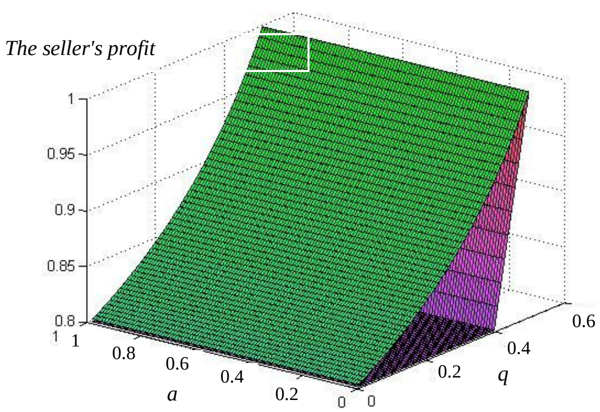
Figure 1



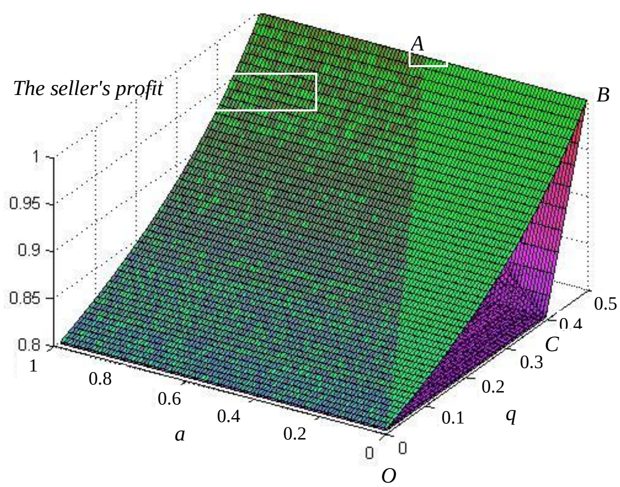
1-1 Seller profit under complete resale



1-2 Seller profit under partial resale(APO) when arbitrage is limited



1-3 Seller profit under partial resale (APO) when arbitrage exists



1-4 The comparison of seller profits under 1-1, 1-2 and 1-3