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Production cost heterogeneity in the circular city model

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ABSTRACT

We derive the closed-form solution that characterizes the equilibrium in the circular city model, when competing firms have heterogeneous production costs. Tractability issues in this setting are well-known and have not been resolved in prior work. In this paper, the equilibrium solution illustrates the effects of production costs on firms' strategic decisions and profits and on consumer surplus.

1. Introduction

The circular city model, based on Salop's influential work [7], is widely applied and analyzed in economics and other related disciplines. Meanwhile, efforts to relax the assumptions of the model are challenged by analytical tractability. An important extension of Salop's work [7] is to examine firm heterogeneity in the spatial context. Many recent studies have contributed to this direction of research [8,1,9,10,2,3]. In [8–10], however, rivals' heterogeneous costs do not play a role in a firm's equilibrium pricing decision. The impact of rivals' heterogeneity on the equilibrium outcome is meaningful because firms' strategic responses to their neighbors' heterogeneous characteristics create a chain-linked effect that propagates throughout the market [4,6], an important phenomenon also observed in practice. Whereas the circular city setup offers the power to capture such an effect, this very feature also substantially complicates the analysis.

This paper allows for the dependency of firms' decisions on rivals' heterogeneous characteristics. The focus is on characterizing the equilibrium pricing decisions of firms with heterogeneous production costs in a circular spatial setting. Our work is closely related to those by Alderighi and Piga [1–3], which also incorporate firms' heterogeneous characteristics that impact their strategic decisions. In this series of studies, Alderighi and Piga identify the uniqueness of equilibrium prices and provide a recursive algorithm for computing the coefficients in the solution. In [1,3], they also present an expression that approximates the solution as the number of firms approaches infinity. However, a general analytical solution that explicitly characterizes the equilibrium outcome is still to be obtained. Using an original approach, our work contributes to this line of fruitful efforts by offering such a solution. It defines each firm's equilibrium price at any given size of the industry and allows clear economic interpretation in the analysis of the competitive equilibrium.

In particular, the closed-form solution enables us to fully explore the role of production cost heterogeneity in the competitive outcome. Production cost heterogeneity causes dispersion in firms' equilibrium prices. But, due to competition, the variance of firms' equilibrium prices is always lower than that of production costs. Moreover, cost heterogeneity lowers the expected average price paid by consumers, though it does not affect the firms' expected equilibrium prices. Further analysis unveils that cost heterogeneity has a positive effect on both the expected aggregate profit and consumer surplus, and, thus, improves the expected social welfare.

Our work builds a theoretical foundation for future applications of the circular city model. This novel approach for deriving the analytical solution is generalizable to circular city settings with heterogeneity other than that in production cost. This facilitates direct analysis of comparative statics that may be useful for yielding managerial and regulatory insights.

2. The model

Our model inherits the standard features of Salop's circular city model [7]. On a circle of unit circumference, a continuum

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of consumers is distributed uniformly, and $n > 1$ firms are located equidistantly. Consumers are indexed by their own locations, which represent taste. Without loss of generality, let firm i be located at $\frac{i}{n}$ and offer products of value v at price p_i. Firms set prices simultaneously. Each consumer purchases at most one unit of product. The distance between a consumer and her chosen firm represents the misfit between the purchased product and her ideal product. Let a consumer's transportation cost be linear in the distance between the firm and the consumer at rate t. Thus, a consumer located at x who purchases from firm i derives the utility $u(x) = v - p_i - t \cdot \left|\frac{i}{n} - x\right|$.

We extend Salop's model [7] to account for heterogeneity in the firms' production costs. Firm i incurs a marginal production cost c_i where $i \in \{0, 1, \ldots, n-1\}$. Following the convention in the literature [5,8,1–3], we examine the equilibrium in which all firms obtain a positive market share. Thus, we impose the following condition throughout the paper to rule out cases in which an existing firm cannot actively compete with the other firms.

Condition 1. $\forall i \in \{0, 1, \ldots, n-2\}, |c_i - c_{i+1}| < \frac{t}{n}$.

The demand for firm i is then $q_i = \frac{1}{n} + \frac{1}{2t}(p_{i+1} + p_{i-1} - 2p_i)$, generating the following profit:

$$\pi_i = (p_i - c_i)\, q_i = (p_i - c_i) \left[\frac{1}{n} + \frac{1}{2t}(p_{i+1} + p_{i-1} - 2p_i)\right]. \tag{1}$$

For notational convenience, we extend the domain of i, such that $i \in \mathbb{Z}$, to allow for continuous increments to firms' indices. Firms i and $i \pm n$ denote the same entity.

3. Equilibrium characterization

Proposition 1. *There exists a unique equilibrium among n firms. For firm i,* $i \in \{0, 1, \ldots, n-1\}$, $p_i^* = \frac{t}{n} + \sum_{d=0}^{n-1} b_d c_{i-d}$,

$$\text{where} \quad b_d = \frac{\left(2+\sqrt{3}\right)^d + \left(2+\sqrt{3}\right)^{n-d}}{\sqrt{3}\left(\left(2+\sqrt{3}\right)^n - 1\right)}. \tag{2}$$

Firm i's profit is $\pi_i^* = \frac{1}{t}\left(p_i^* - c_i\right)^2$.

Proof. All proofs are provided in the supplementary content (see Appendix A).

Proposition 1 summarizes an important contribution of this work—the closed-form solution of the equilibrium prices in the circular city model with cost heterogeneity. The functional form of p_i^* shows that each firm's pricing strategy depends not only on its own production cost but also on the production costs of all the other firms in the market. A firm's pricing strategy and profit are directly affected by its first-degree neighboring rivals, located to its immediate left and right. The first-degree neighboring rivals also compete directly with their neighbors, who are second-degree neighbors to the original firm. Competition propagates around the circle in this matter and links all firms' pricing strategies together. This coincides with the chain-linked effect discussed in the previous literature [4,6].

As a result of the chain-linked effect, the coefficients, b_d's, analytically quantify the impact of heterogeneous production costs on each firm's equilibrium price and profit. Thus, b_1 is the magnitude of the direct impact from a first-degree neighbor, whereas b_d ($d > 1$) represents the indirect impact originated from a firm located further away with d degrees of separation from the affected firm.

Corollary 1. (a) $b_d = \frac{\partial p_i^*}{\partial c_{i-d}} = \frac{\partial p_{i\pm d}^*}{\partial c_i} > 0$, *for* $d = 0, 1, \ldots, n-1$. *An increase in any firm's production cost leads to a higher equilibrium price for all firms; moreover, this impact only depends on the degree of separation between the two firms.*

(b) $b_d > b_{d'}$, *if* $\min\{d, n-d\} < \min\{d', n-d'\}$. *The effect of production cost weakens as it reaches firms that are further away from the originating firm.*

Based on p_i^* from Proposition 1, $b_d = \frac{\partial p_i^*}{\partial c_{i-d}}$ represents the impact of the production cost of firm i's dth-degree neighbor on firm i's price. Alternatively, $b_d = \frac{\partial p_{i\pm d}^*}{\partial c_i}$, which is the impact of firm i's production cost on its dth-degree neighbor's price. The subscript $i \pm d$ refers to the dth-degree neighbor on either left or right side of firm i. This shows that the impact of a firm's production cost on another firm is a positive constant that only depends on the number of firms this impact traverses through to reach the affected firm. Intuitively, an increase in a firm's production cost leads to a higher price for the firm itself; at the same time, such a price increase mitigates the firm's price competition with its first-degree neighbors, who also raise their prices. The incentive to raise prices is then transferred from firm to firm around the circle, resulting in price increases for all firms.

Furthermore, the expression of b_d (Eq. (2)) analytically shows that the impact of a firm's production cost on other firms' prices weakens quickly as it travels further away from the originating firm. This implies that a firm's price is affected more strongly by the production costs of the firms that offer more similar products and, in fact, *primarily* by the firm's first- and second-degree neighbors. This is also illustrated in [2] with numerical approximations (see Table 1 on p. 53 of [2]). Our result provides an analytical validation based on the explicit solution of b_d (Eq. (2)), which generates values (as shown in Table 1) that are consistent with those computed by Alderighi and Piga in [2].

Corollary 2. *As a firm's production cost increases, only a part of the cost increase is absorbed by the firm's price increase, i.e.,* $0 < b_0 < 1$. *The remaining cost increase is fully captured by the rest of the firms in the increments of their equilibrium prices, i.e.,* $\sum_{d=0}^{n-1} b_d = 1$.

Competition moderates the extent of a firm's price increase that results from a higher production cost. Interestingly, the part of the cost increase that is not absorbed by the firm's price is shared by the remaining firms in their equilibrium prices. In fact, any change in a firm's production cost is fully accounted for in the equilibrium prices of the entire market. Firms are able to completely transfer this shift in cost to consumers, not only to those buying from the firm that incurs the cost shift, but also to those buying from the competing firms.

Corollary 3. *An increase in any firm's production cost reduces its own profit (i.e.,* $\frac{\partial \pi_d^*}{\partial c_d} < 0$*) and generates higher profits for the competing firms (i.e.,* $\frac{\partial \pi_{d'}^*}{\partial c_d} > 0$, $\forall d' \neq d$*).*

Expectedly, an increase in a firm's production cost leads to a loss in that firm's profit. Due to competition, the firm's price cannot fully absorb its cost increase, and its market share is reduced as a result of the higher price. On the other hand, the other firms in the market benefit from the mitigated price competition and gain higher profits.

4. Analysis: price, profit, and consumer surplus

We now turn to further analysis of the expected equilibrium price, aggregate profit and consumer surplus according to a general distribution of production cost subject to Condition 1. Let us consider an industry in which firms' production costs are independent of each other and follow an identical cumulative distribution function, $F(c_i)$.

Table 1
Numerical values of b_d based on Eq. (2).

	$d = 0$	$d = 1$	$d = 2$	$d = 3$
$\lim_{n\to\infty} b_d$	$\frac{1}{\sqrt{3}} \approx 0.5774$	$\frac{1}{\sqrt{3}(2+\sqrt{3})} \approx 0.1547$	$\frac{1}{\sqrt{3}(2+\sqrt{3})^2} \approx 0.0415$	$\frac{1}{\sqrt{3}(2+\sqrt{3})^3} \approx 0.0111$

Proposition 2. (a) *A firm's expected equilibrium price, $E\left(p_i^*\right)$, consists of the expected production cost, $E\left(c_i\right)$, and a constant markup:*

$$E\left(p_i^*\right) = E\left(c_i\right) + \frac{t}{n}. \tag{3}$$

(b) *The expected price paid by consumers (equivalently, the expected demand-weighted average price), $E\left(\tilde{p}_i^*\right)$, is:*

$$E\left(\tilde{p}_i^*\right) = E\left(p_i^*\right) - \frac{n}{t}Var\left(c_i\right) \times \left(\frac{\left(2+\sqrt{3}\right)^n + 1}{3\sqrt{3}\left(\left(2+\sqrt{3}\right)^n - 1\right)} - \frac{2n\left(2+\sqrt{3}\right)^n}{3\left(\left(2+\sqrt{3}\right)^n - 1\right)^2}\right), \tag{4}$$

which decreases in the variance of production cost.

Proposition 2(a) indicates that any shift in the expected value of the firms' production costs is completely transferred to the expected equilibrium price (Eq. (3)). This insight closely follows Corollary 2 and shows that any change in production cost is fully captured by the total price adjustments of all firms. Furthermore, the expected markup is constant: A firm's expected equilibrium price decreases when more firms are in the market (a higher n) due to competition and increases when consumers' transportation cost t is higher, which mitigates competition.

By weighting each firm's equilibrium price by its market share, we derive the expected equilibrium price paid by consumers, which can also be interpreted as the expected price per product. Proposition 2(b) shows that the expected equilibrium price paid by consumers is lower than the expected equilibrium price. This is intuitive because lower-cost firms set lower equilibrium prices and hold larger market shares (i.e., lower prices are weighted more than higher prices). As firms' production costs become more heterogeneous, more products are sold at lower prices, and fewer are sold at higher prices. Thus, the gap between these two prices increases with the variance of production cost. Moreover, when more firms are in the market, or when consumers' transportation cost is lower, firms compete more intensely. Consequently, cost heterogeneity has a more pronounced effect on the equilibrium market shares, which widens the gap between the two expected prices.

Proposition 3. (a) *The variance of firms' equilibrium prices, $Var(p_i^*)$, increases in the variance of the firms' production costs, $Var(c_i)$; furthermore, $Var(p_i^*) < Var(c_i)$.*

(b) *The expected aggregate profit of all firms is:*

$$E\left(\sum_{i=0}^{n-1}\pi_i^*\right) = \frac{t}{n} + \frac{n}{t}Var\left(c_i\right)\left(1 - \frac{4}{3\sqrt{3}} \cdot \frac{\left(2+\sqrt{3}\right)^n + 1}{\left(2+\sqrt{3}\right)^n - 1} + \frac{2n\left(2+\sqrt{3}\right)^n}{3\left(\left(2+\sqrt{3}\right)^n - 1\right)^2}\right). \tag{5}$$

Not surprisingly, production cost heterogeneity causes dispersion in firms' equilibrium prices; however, the variance of the firms' equilibrium prices is always lower than that of the firms' production costs. An increase in the variance of production cost allows higher-cost firms (which are also firms with higher prices) to charge more and lower-cost firms to charge less, because a firm's equilibrium price increases in its own cost (Corollary 1(a)). The variance of firms' equilibrium prices in turn increases. Meanwhile, all firms' equilibrium prices shift in the same direction in response to the changes in production costs. This distributed price change tightens variation in price relative to that in production cost.

Furthermore, shifts in the expected value of production cost has no impact on competition intensity if the variance of the product cost distribution remains constant; thus, the expected aggregate profit is unaffected by the expected value of production cost but increases in the variance of production cost (Eq. (5)). This is consistent with the discussion in [3]. An increase in the variance of production cost leads to a higher degree of asymmetric competition. Given that $0 < b_0 < 1$ (Corollary 2), the firms with increased costs suffer both lower margins and smaller market shares, whereas the opposite applies for the firms with reduced costs. Therefore, as the variance of production cost increases, the proportion of market transactions with an increased margin outweighs that with a reduced margin. As a result, the expected aggregate profit grows.

Proposition 4. *The expected total transportation cost is*

$$E\left(\tau\right) = \frac{t}{4n} + \frac{nVar\left(c_i\right)}{6t\left(\left(2+\sqrt{3}\right)^n - 1\right)^2} \times \left(\frac{\left(2+\sqrt{3}\right)^{2n} - 1}{\sqrt{3}} - 2n\left(2+\sqrt{3}\right)^n\right), \tag{6}$$

which is increasing in the variance of production cost. The expected consumer surplus is $E\left(u\right) = v - E(c_i) - \frac{5t}{4n} + \frac{nVar(c_i)}{6t\left(\left(2+\sqrt{3}\right)^n-1\right)^2}\left(\frac{(2+\sqrt{3})^{2n}-1}{\sqrt{3}} - 2n\left(2+\sqrt{3}\right)^n\right)$, which is increasing in the variance of production cost and decreasing in the expected production cost.

A higher variance of production cost not only raises the aggregate profit (Proposition 3(b)), but it also increases the expected consumer surplus despite an increased total transportation cost. When firms' production costs are more heterogeneous, their equilibrium prices also become more dispersed (Proposition 3(a)). In expectation, consumers are more inclined to purchase from firms with lower prices while bearing higher transportation costs. In aggregate, the reduction in the expected price paid by consumers due to the increased variance of production cost (Proposition 2(b)) outweighs the increased transportation cost; thus, the expected consumer surplus is higher with more heterogeneous production costs.

Propositions 3 and 4 imply that an increasing cost variance while fixing the expected cost results in higher social welfare. This result suggests a potential mechanism to increase the profitability of an industry without taxing consumers. Whereas overall improvements in production efficiency may actually intensify competition between firms, balanced shocks, such as policies to regulate supplier contracts to induce dispersion of production efficiencies or capabilities, may improve social welfare.

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Appendix A. Supplementary data

Supplementary material related to this article can be found online at http://dx.doi.org/10.1016/j.orl.2015.04.010.

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