Chapter V. Information and Asset Pricing

By assumption in the CAPM information is costlessly available and is interpreted identically by all individuals. As a result, expectations are homogeneous and, given additional perfect markets assumptions, the perceived investment opportunity sets are the same for all investors. In this chapter we drop the assumption that information is costless, while maintaining the assumption that a given piece of information is interpreted identically by all and is considered rationally by all.

1. MARKET EFFICIENCY

The rational use and interpretation comes into play when we try to define the notion of an "efficient market." In general terms, a market is efficient when no obvious profit opportunities are available, when there are "no $100 dollar bills lying in the street." This concept of an efficient market is wholly unrelated, by the way, to the concept of an "efficient market portfolio" as was used in the CAPM context. The latter refers to the, given its mean, presumed minimum variance property of the market portfolio; the former refers to the presumed absence of easy profit opportunities. The APT is based on market efficiency in the sense that, by assumption, no arbitrage profits are available.

A precise and operational definition of market efficiency requires reference to the information set that an investor uses to infer profit opportunities. Fama (1970, p. 383) defines market efficiency in general as follows: "A market in which prices 'fully reflect' available information is called 'efficient'." In such a market, clearly, no easy profit opportunities remain. To operationalize the definition Fama distinguishes three forms by what type of information is assumed to be available. *Weak form efficiency* takes the available information to be just historical prices; *semi-strong form efficiency* takes the information set to be any information that is publicly available; *strong form efficiency* concerns an even larger information set, namely the information available to any group of investors. In this last case, for example, a market would be inefficient if it is found that some piece of information, available at some time t to some group of investors, could be exploited at a later time $t + i$.

Fama proposes the "efficient market hypothesis" according to which it should not be possible to devise trading rules, using available (depending on which of the three definitions is used: past price, public, or private) information, that allow systematic profits to be made over and above transaction costs and a proper compensation for risk. Slightly more formally Jensen (1978, p. 96) defines: "A market is efficient with respect to information set ω_t, if it is impossible to make profits on the basis of information set ω_t." Fama's (1970) survey of the literature concludes that, on the whole, markets are efficient under all three of the information assumptions.

In an update of his market efficiency survey, Fama (1991, p. 1575) admits that the strong form of market efficiency requires that information and trading costs – the costs of getting prices to reflect the information, be always zero. He agrees with Jensen (1978) that an economically more sensible version of the efficiency hypothesis says that prices reflect information to the point where the marginal benefits of acting on information (the profits to be made) do not exceed the marginal costs. An article by Grossman and Stiglitz (1980) to be discussed in the following clarifies this issue.

Fama argues, however, that information and trading costs are not the biggest hurdle in testing market efficiency. Instead, a more serious problem is that of testing *jointly* market efficiency with a particular equilibrium asset-pricing model: "...we can only test whether information is properly reflected in prices in the context of a pricing model that defines the meaning of 'properly'." Thus, a model is necessary to dictate how to adjust for risk and to indicate what is relevant information. As a result, when empirical results indicate abnormal profits, it can always be argued that the asset pricing model used is false in the sense that the abnormal profits are due to some risk factor that is ignored in the pricing model.

In the updated survey, Fama now considers general tests for return predictability instead of weak form market efficiency; event studies (in response to publicly revealed information) instead of semi-strong market efficiency; and tests for private information instead of strong form market efficiency. While evidence exists against market efficieny, Fama argues that the event studies on the whole tend to support market efficiency. This is significant since event studies mostly deal with short time windows so that the influence of the asset pricing model chosen to correct for risk is minimal and we have the purest test of the market efficiency hypothesis in isolation.

More recently, Fama (1998) adds an additional argument in favor of market efficiency. He notes that market efficiency appears often to be rejected in the literature. However, there is little system in the rejections. For instance, some studies find that prices overreact to public information; others find that prices underreact to public information. Fama argues that if anomalies split randomly between overreaction and underreaction, they are consistent with market efficiency. He also argues that many of the empirical anomalies discovered are not robust to small changes in the methods used to establish them. In general, whether markets are efficient or not is heavily debated. In the end one may have to decide whether the glass is half full or half empty. At a minimum, the market efficiency assumption is quite descriptive of reality in many cases. In the following we will maintain the basic notion of market efficiency while arguing against the strong form of market efficiency.

The general definition of efficient markets, in principle, accounts for the fact that information may be costly to obtain (or that transactions may be costly). Thus, if private information could have been used to generate abnormal profits this is not in itself evidence against market efficiency: most investors might have chosen rationally in advance not to become informed to avoid the information cost. Grossman and Stiglitz (1980) note a problem with the definition of market efficiency in this context. If information is costly to obtain and if prices always fully reflect all relevant information, then no investor has an incentive to become informed. One might just observe market prices and effectively glean all relevant information without incurring the cost. But, clearly then nobody will spend the resources to become informed, and prices cannot reflect information that nobody possesses.

Grossman and Stiglitz's answer is that the semi-strong form of market efficiency should hold: prices reflect all relevant information that is publicly available. Prices only partly reflect relevant private information. Informed bid up (down) prices based on their private positive (negative) information. The analysis of their model below explains why uninformed investors cannot use the price information to deduce the information of the informed investors and why apparent profit opportunities remain.

2. THE GROSSMAN AND STIGLITZ MODEL

Grossman and Stiglitz (1980) assume that all investors are basically identical. However, some choose to become informed (at cost c) and some choose to remain uninformed. Each investor is risk averse and has constant absolute risk aversion (CARA) preferences; wealth may differ across individuals but this makes little difference due to the CARA assumption. Only two financial assets exist, one risky and one riskless. Apart from the cost of information acquisition, markets are perfect. Investors are fully rational. The model is a static one-period model (as is the CAPM).

(a) The model

The "true" value of the risky asset is given as:

(1) $$u = \theta + \varepsilon,$$

where u reflects the actual (liquidation) dividends paid out to the investor at the end of the period; θ can be observed a priori at a cost c ; ε is unobservable a priori and reflects a random shock to the firms dividends with mean zero. The shock ε is important in the model as it implies that even investors that choose to become informed will still face some return risk if the purchase the asset.

The budget constraint of any investor i, where i may represent either an informed investor I or an uninformed investor U, is given as:

(2) $$p X_i + M_i = \bar{w}_i.$$

Here p indicates the price of the risky asset (thus the gross investment return will be u/p). X_i is the quantity of the risky asset demanded by investor i ; M_i is the quantity demanded of the riskless asset. Initial wealth of investor i is given by $\bar{w}_i$.

Final wealth w_i is fully consumed by each investor and is given as:

(3) $$w_i = u X_i + R M_i - c_i,$$

with R the gross riskless return, which is given exogenously. $c_I = c$ ($i = I$, the investor is informed) and $c_U = 0$ ($i = U$, the investor is uninformed). The utility of consumption and thus final wealth is represented by

(4) $$v(w_i) = -e^{-a w_i},$$

where $a > 0$ indicates the constant of absolute risk aversion.

The riskless asset is inelastically supplied; the supply of the risky asset per investor is unobservable until the end of the period and is random and equal to x. There are therefore three random variables in the model which are assumed to be normally distributed and mutually uncorrelated:

$$(5) \qquad \theta \sim N(\bar{\theta}, \sigma_\theta^2)\ , \quad \varepsilon \sim N(0, \sigma_\varepsilon^2)\ , \quad x \sim N(\bar{x}, \sigma_x^2)\ .$$

The randomness in the supply per investor of the risky asset x is important in the model in order to confuse the uninformed investors about the information held by the informed investors: if price increases this may be due either to favorable information about the value of the asset θ or it may be just due to a random reduction in the supply of the asset x. A realistic motivation for a random (net) supply of the risky asset is that individuals are often forced to sell assets for liquidity purposes that are investor specific, independently of what the asset is worth. The aggregate number of assets sold by "liquidity traders" can therefore be thought of as a random increase in the available supply of the asset.

(b) Portfolio choice

We first consider the model taking the decision to become informed as already made previously. All investors maximize their expected utility. Given the normality assumption and the CARA utility function, equations (4) and (5) give:

$$(6a) \qquad E(-e^{-a w_i}) = -e^{-a[E(w_i) - (a/2)\sigma_{w_i}^2]}.$$

Note that this equality is very useful in many applications which assume CARA utility and normally distributed random variables. A monotonic transformation (that we know, from standard micro, does not affect optimal choice) then provides a simple form for the investor's objective:

$$(6b) \qquad \underset{X_i}{\text{Max}} \quad E_i(w_i) - (a/2)\,\sigma_{w_i}^2\ .$$

The subscript i under the expectations operator E indicates that expectations are formed rationally based on the information available to investor i. We find for each investor, after eliminating the budget constraint:

$$(7) \qquad E_i(w_i) = R(\bar{w}_i - pX_i) + E_i(u)\,X_i - c_i\ ,$$

and

$$(8) \qquad \sigma_{w_i}^2 = X_i^2\,\sigma_{u_i}^2\ ,$$

where again the subscript i, here attached to the conditional variance, indicates that the variance is taken subject to the information available to investor i.

The decision problem of equation (6b) subject to equations (7) and (8) now implies after taking the derivative with respect to X_i :

$$(9) \qquad X_i^* = \frac{E_i(u) - Rp}{a\,\sigma^2_{u_i}} .$$

Thus investment in the risky asset depends on the expected payoff from the risky asset net of the opportunity cost (the payoff of investing in the riskless) divided by the disutility of the risk inherent in the risky asset; the latter being the product of the investor's degree of risk aversion and the quantity of risk as measured by the investor's conditional variance of the payoff.

For the *informed* investor, equation (9) becomes:

$$(10) \qquad X_I^* = \frac{\theta - Rp}{a\,\sigma^2_{\varepsilon}} ,$$

since, from equation (1), the expected payoff from the risky asset equals $E_i(\theta)$, which is known to the informed investor and equal to θ, plus $E(\varepsilon)$, which is zero for all investors; and since the variance of the risky payoff given that θ is known is equal to σ^2_{ε}. For the *uninformed* investor, equation (9) becomes:

$$(11) \qquad X_U^* = \frac{E(\theta|p) - Rp}{a\,(\sigma^2_{\theta|p} + \sigma^2_{\varepsilon})} ,$$

with the information of the uninformed investors limited to the observable market price. Accordingly, $E_I(u) = E(u|p) = E(\theta|p) + E(\varepsilon|p)$, with the last term *0* since the random dividend component is unknown to all; the conditional variance of the risky payoff is obtained similarly and appears as the term in parentheses in the denominator.

(c) Rational expectations model solution

The equilibrium market price of the risky asset is determined by demand and supply:

$$(12) \qquad \lambda X_I^* + (1 - \lambda) X_U^* = x,$$

where λ is the fraction of investors that is informed. (Recall that x represents the random asset supply per investor). We now find the equilibrium market price p^* in rational expectations market equilibrium as that price p^* determined such that no investor wants to recontract given the information obtained by observing p^*. A step-by-step solution of the rational expectations equilibrium employing the *method of undetermined coefficients* is as follows:

Step 1. Consider that the market equilibrium condition, equation (12), with equations (10) and (11) substituted in, must hold as an identity for the equilibrium price p^* :

$$\lambda\left(\frac{\theta - Rp^*}{a\,\sigma_\varepsilon^2}\right) + (1-\lambda)\left(\frac{E(\theta|p^*) - Rp^*}{a\,(\sigma^2_{\theta|p^*} + \sigma_\varepsilon^2)}\right) \equiv x. \tag{13}$$

Step 2. Assume an equilibrium price that is linear in the random variables with the coefficients α_i as yet undetermined:

$$p^* = \alpha_1\theta + \alpha_2 x + \alpha_3. \tag{14}$$

Note that there is a coefficient for all random variables as well as a constant, except for the ε term since no possible information regarding ε can possibly occur during the period to affect either demand or supply.

Step 3. For the postulated pricing equation, find $E(\theta|p^*)$ and $\sigma^2_{\theta|p^*}$. Bayesian updating, given the normality of the variables involved, yields that

$$E(\theta|p^*) = \bar{\theta} + (\sigma_{\theta p^*}/\sigma^2_{p^*})\,[p^* - E(p^*)],$$

that is, the expectation of θ based on observing the equilibrium price is equal to its unconditional expectation $\bar{\theta}$ plus the fraction of the surprise in the equilibrium price observation that can be attributed to the information about θ available to the informed. This fraction is determined as the simple slope of regressing θ on p^*. From equation (14) we have that

$$\frac{\sigma_{\theta p^*}}{\sigma^2_{p^*}} = \frac{\alpha_1\sigma_\theta^2}{\alpha_1^2\sigma_\theta^2 + \alpha_2^2\sigma_x^2} \quad\text{and}\quad p^* - E(p^*) = \alpha_1(\theta - \bar{\theta}) + \alpha_2(x - \bar{x}).$$

Combining these equations produces:

$$E(\theta|p^*) = \bar{\theta} + \frac{\alpha_1^2\sigma_\theta^2}{\alpha_1^2\sigma_\theta^2 + \alpha_2^2\sigma_x^2}(\theta - \bar{\theta}) + \frac{\alpha_1\alpha_2\sigma_\theta^2}{\alpha_1^2\sigma_\theta^2 + \alpha_2^2\sigma_x^2}(x - \bar{x}). \tag{15}$$

We can use equation (15) to obtain $\sigma^2_{\theta|p^*} \equiv E[\theta - E(\theta|p^*)]^2$:

$$\sigma^2_{\theta|p^*} = \sigma_\theta^2 - \frac{\alpha_1^2\sigma_\theta^4}{\alpha_1^2\sigma_\theta^2 + \alpha_2^2\sigma_x^2} = \left(\frac{\alpha_2^2\sigma_x^2}{\alpha_1^2\sigma_\theta^2 + \alpha_2^2\sigma_x^2}\right)\sigma_\theta^2. \tag{16}$$

Step 4. Substitute equations (14), (15) and (16) into equation (13). Then pick the α_i such that the resulting equation holds as an identity. This yields:

$$\text{(17a)} \qquad \alpha_1 = [\lambda(\sigma_\varepsilon^{-2} + \sigma_\theta^{-2}) + (a\sigma_\varepsilon^2/\lambda)^{-2}\sigma_x^{-2}]/\gamma R > 0\,,$$

$$\text{(17b)} \qquad \alpha_2 = -(a\sigma_\varepsilon^2/\lambda)\,\alpha_1 < 0,$$

$$\text{(17c)} \qquad \alpha_3 = (1-\lambda)[\sigma_\theta^{-2}\bar{\theta} + (a\sigma_\varepsilon^2/\lambda)^{-1}\sigma_x^{-2}\bar{x}]/\gamma R\,,$$

with

$$\text{(17d)} \qquad \gamma \equiv \lambda(\sigma_\varepsilon^{-2}) + \sigma_\theta^{-2} + (a\sigma_\varepsilon^2/\lambda)^{-2}\sigma_x^{-2}.$$

Thus, the rational expectations solution of the equilibrium market price is confirmed and given by equation (14) with the values for the coefficients provided in equation (17).

(d) Interpretation

The sign of α_2 is negative. From equation (14) this is sensible since a positive supply shock x should have a negative impact on the market price. Similarly, the positive sign of α_1 makes sense in equation (14) as it implies that positive fundamental information about the asset (a higher θ) raises the price of the asset.

Rewriting equation (14) based on equation (17b) gives:

$$\text{(18)} \qquad p^* = \alpha_1[\theta - (a\sigma_\varepsilon^2/\lambda)(x - \bar{x})] + \alpha_3 + \alpha_2\bar{x}.$$

Since the α_i and $\bar{x}$ are known constants, observing p^* is easily deduced to be the same as observing:

$$\text{(19)} \qquad w(\lambda) = \theta - (a\sigma_\varepsilon^2/\lambda)(x - \bar{x}),$$

which is in essence a noisy observation on θ. If the last term in equation (19) is zero the market price fully reveal the information collected by the informed investors. Hence, *if and only if* a or σ_ε^2 or σ_x^2 equals zero is the market price fully informative about θ (note that the case of $\lambda \to \infty$ can be ruled out since $\lambda \leq 1$).

Intuitively, if a is zero, all investors, and informed investors in specific, are risk neutral. Thus, if the market price differs from θ (the expected market price given that θ is known) the risk neutral informed investors will continue to buy or sell the risky asset until the difference disappears. In the end we must have $p^* = \theta$ so that the market price is fully revealing. If σ_ε^2 is zero, investors need not be risk neutral but the risk of investing in the risky asset becomes zero for all informed investors, since from equation (1) they now know the actual dividend that the risky asset will pay. Hence, again the informed investors will keep buying or selling the risky asset until $p^* = \theta$. If σ_x^2 is equal to zero, there is no supply uncertainty. Given that investors know the structure of the model, they know that any change in price can be due only to the content of the information received by the informed investors; consequently there is a monotonic and deterministic relation between p^* and θ that the uninformed can exploit to figure out the value of θ derived from the demand by the informed.

From equation (19) it is straightforward to obtain some comparative statics results. If a or σ_ε^2 falls then informed investors are more responsive to changes in θ because risk is less important. So that their demands are more heavily reflected in the price, making the price more informative about θ. If σ_x^2 falls there is less "non-fundamental" noise to block the "view" of the uninformed investors. Thus price is more revealing about θ. Lastly, if λ increases, a larger fraction of the investors is informed so that their demand has more impact on the price, making price again more revealing about θ.

(e) The equilibrium fraction of informed investors

The equilibrium thus far was derived for a given proportion of informed investors λ. The choice to become informed, however, is endogenous and depends on the maximum expected utility of wealth after information acquisition costs when informed $E[v(w_I^*)]$, relative to the maximum expected utility of wealth when uninformed $E[v(w_U^*)]$. If $E[v(w_I^*)]$ > $E[v(w_U^*)]$ then the fraction of informed investors λ will increase; and vice versa. When λ increases (falls), the price becomes more (less) revealing about θ (as discussed in the previous sub section) thus the benefit of becoming informed falls (increases). Hence, in equilibrium:

$$(20) \qquad E[v(w_I^*)] = E[v(w_U^*)] \ ,$$

and this equation determines the equilibrium fraction of informed investors λ^*. Note that the decision to acquire information is made before θ or p^* is observed. In general information benefits the investor in two ways: (a) it allows him to adjust demand to increase expected return and (b) his risk after demand adjustment is reduced.

Equation (6a) for the uninformed investor together with equation (11) yields:

$$(21) \qquad E[v(w_U^*)|p] = -e^{-aR\bar{w}} \ e^{-[E(\theta|p) - Rp]/[a(\sigma_{\theta|p}^2 + \sigma_\varepsilon^2)]},$$

Note that the expected utility is conditional on some price p which is not known at the time of the information acquisition decision.

Similarly, the expected utility, conditional on some price p which is not known at the time of the information acquisition decision, for an investor who is informed would be given from equations (6a) and (10) as

$$E[v(w_I^*)|p] = -e^{-aR(\bar{w}-c)} \ E\left(e^{[-(\theta - Rp)^2/2(\sigma_\varepsilon^2)]} \ | \ p\right).$$

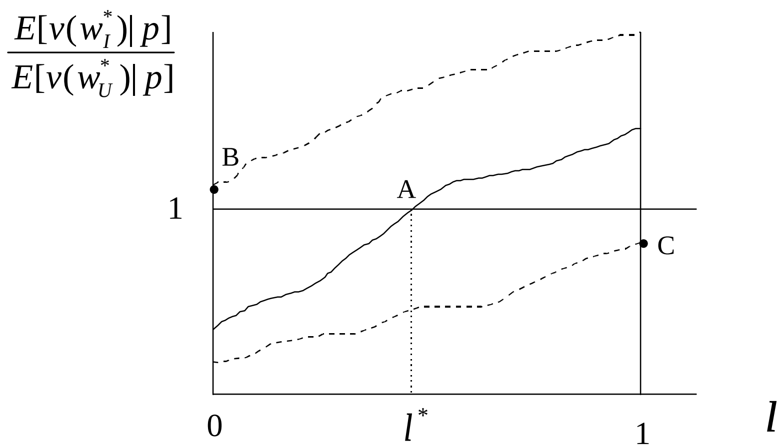

Figure 1

The equilibrium fraction of informed investors

Point A indicates an interior equilibrium; points B and C indicate corner solutions. The graphs slope upward, indicating that becoming informed is less attractive the more investors are already informed

The expectations operator cannot be pulled through easily here because θ is random and the term in the exponent does not have the normal distribution. However, the exponent is the square of a normally distributed variable and has a non-central χ^2 distribution. Appendix B in Grossman and Stiglitz (1980) shows that we can write the above equation as:

$$(22) \qquad E[v(w_I^*)|p] = -e^{-aR(\bar{w}-c)} \, [\sigma_\varepsilon/(\sigma^2_{\theta|p} + \sigma^2_\varepsilon)^{1/2}] \; e^{-[E(\theta|p) - Rp]/[a(\sigma^2_{\theta|p} + \sigma^2_\varepsilon)]}.$$

Comparing equations (21) and (22) shows that

$$(23) \qquad E[v(w_I^*)|p] = e^{ac} \, [\sigma_\varepsilon/(\sigma^2_{\theta|p} + \sigma^2_\varepsilon)^{1/2}] \; E[v(w_U^*)|p].$$

Now, if

$$(24) \qquad e^{ac} \, [\sigma_\varepsilon/(\sigma^2_{\theta|p} + \sigma^2_\varepsilon)^{1/2}] = 1,$$

taking unconditional expectations in equation (23) directly yields equation (20) so that the equilibrium fraction of informed investors λ^* can be obtained from equation (24). Given rational anticipation, the term $\sigma^2_{\theta|p}$ can be found by dividing numerator and denominator in equation (16) by α_1^2 and using equation (17b):

$$(25) \qquad \sigma^2_{\theta|p} = \frac{\sigma_x^2\,\sigma_\theta^2}{(\lambda / a\,\sigma_\varepsilon^2)^2\,\sigma_\theta^2 + \sigma_x^2}\,,$$

with the α_i stated in equations (17a) and (17b).

Conducting comparative statics analysis on equations (24) and (25) yields the following results. First consider that λ and $\sigma^2_{\theta|p}$ are inversely related as follows from equation (25). This is akin to finding that more informed investors implies that p becomes more informative about θ. Hence we know that the relative benefit of being informed *falls* as the proportion of informed investors *rises*. Figure 1 illustrates this fact (in considering Figure 1, note that utility is always negative in this model so that an increase in $|E[v(\cdot)]|$ lowers expected utility!) Figure 1 also shows that it is not necessary that an interior equilibrium λ^* exists: if the expected benefit of being informed is less than that of being uninformed then λ will fall this increases the relative benefit of being informed but maybe not quick enough: if the expected utilities are not equal when
$\lambda = 0$ then no interior equilibrium exists and the outcome $\lambda = 0$ will prevail so that no investor chooses to become informed. Vice versa, if initially the expected benefit of being informed is more than that of being uninformed then λ will rise, this lowers the relative benefit of being informed but maybe not quick enough, so that no interior equilibrium exists and all become informed $\lambda = 1$.

For the sake of comparative statics analysis, however, we assume that an interior equilibrium λ^* exists. Consider an increase in the information acquisition cost c; from equations (23) and (24) the graph in Figure 1 shift up, causing λ^* to fall: if the information acquisition cost is higher, in equilibrium fewer investors will choose to become informed. If σ_x^2 increases then $\sigma^2_{\theta|p}$ falls for given λ. Thus, the graph in Figure 1 shifts down and more investors choose to become informed; λ^* rises. The reason is that it is now harder to infer useful information from the equilibrium price alone so that being uninformed is less attractive. From equation (25), an increase in σ_θ^2 requires λ^* to rise. This happens because the higher variability in θ makes acquiring information more useful.

An increase in the constant of absolute risk aversion a has an ambiguous effect on λ^*: on the one hand, more risk aversion raises the opportunity cost of giving up the certain cash flow c (lowering λ^*); on the other hand, more risk aversion implies that informed investors react less strongly to θ so that the price is less informative (raising λ^*). Similarly, an increase in the random price component's variability σ_ε^2 increases the risk to informed investors so that price becomes less informative (raising λ^*); but an increase in σ_ε^2 also reduces the demand of uninformed investors and provide less of an incentive for investors to become informed because they will not be able to react to their information as aggressively (lowering λ^*).

(f) Discussion

One question to ask with this model, is how informative prices are in equilibrium. There are various ways of measuring the degree of information contained in the price. A reasonable measure is $1/\sigma_{u|p}$, the inverse of the standard deviation of the value of the asset conditional on observing the market price. It follows directly from equation (24) that:

(26) $$1/\sigma_{u|p} = \sigma_\varepsilon / e^{ac} .$$

Thus price is more informative as σ_ε is higher and as a and c are lower. More surprising is that the equilibrium informational content does not change with σ_θ^2 or with σ_x^2. The reason for, say, an increase in σ_x^2 is that expected utility for the uninformed decreases. Hence, λ^* increases. This continues until profits are equalized, which implies from equation (24) that

$\sigma_{u|p}$ must be unchanged. A similar argument applies for why σ_θ^2 does not affect the equilibrium information content of price.

A natural follow-up question then is: Does the market imply under-investment in information? The answer is yes. Investors do not internalize the positive externalities that their information acquisition imposes on the uninformed investors; similarly, the uninformed investors have an incentive to free-ride on the information acquired by the informed, which reduces the risk of the uninformed.

Another issue is the one that Grossman and Stiglitz raise in the introduction of their paper: under what circumstances may an equilibrium fail to exist? Intuition tells us that an equilibrium may not exist if nobody has an incentive to become informed, because then the incentive to become informed should be there anyway. (Recall Yogi Berra's saying: "Nobody goes there no more since it is too crowded."). Suppose that σ_x^2 or σ_ε^2 equals zero and that $\lambda > 0$. Then $\sigma_{u|p} = \sigma_\varepsilon^2$ since from equation (19) market price is perfectly informative about θ. Accordingly, from equation (24) we have $e^{ac} = 1$, but $e^{ac} > e^0 = 1$ which implies a contradiction. Then it must be for equilibrium to exist that $\lambda = 0$. In this case we know that $\sigma_{\theta|p} = \sigma_\theta$. Thus, from equation (24): $e^{ac}\ [\sigma_\varepsilon / (\sigma_\theta^2 + \sigma_\varepsilon^2)^{1/2}] = 1$. If $\sigma_\varepsilon^2 = 0$ then this equation implies $0 = 1$; for $\sigma_x^2 = 0$ the contradiction is not as obvious but it is clear that if parameters are such that $e^{-ac} > [\sigma_\varepsilon / (\sigma_\theta^2 + \sigma_\varepsilon^2)^{1/2}]$ no equilibrium exists. Related arguments can be made as well to show that for $a = 0$ or for $c = 0$ (use Figure 1) no equilibrium may exist.

Equilibrium may also fail to exist if a secondary market for information is added to the model. That is, if informed investors are allowed to sell their information. Since the marginal cost of selling the information in the secondary market would be zero, the acquisition cost of information would be competed down to zero so that, either no equilibrium would exist since nobody wants to become informed in the primary market at cost c, or one investor becomes informed and acts as a monopolist in the secondary market. One factor that argues against the importance of this secondary market issue is that information sold in secondary markets may not be credible.

A further factor that may affect existence of equilibrium in the Grossman-Stiglitz model is the presence of a futures market in the risky asset. If such a futures market exists, uninformed investors observe an additional price signal which they can exploit to identify the sources of price variation. With both a spot market price signal and a futures market price signal, uninformed investors may be able to disentangle the non-fundamental supply shock x and the fundamentals shock θ so that again prices become fully informative and an equilibrium does not exist. A solution to this would be to introduce an additional random shock to ensure that the number of unobservable random variables exceeds the number of price signals.

A final criticism of the Grossman and Stiglitz model is that it does not allow any dynamics. In practice, information acquisition is more useful the earlier the information is obtained since the investor who gets the information first can typically trade at prices that are the furthest removed from the "informed" price.

(g) Applications and Exercises

1. For the Grossman and Stiglitz (1980) paper, answer the following questions.

(a) Explain in words why "strong-form" market efficiency, in the sense that financial market prices reflect all relevant information, is impossible when obtaining information is costly.

(b) Explain in words and using some of the equations of the model why it is essential in the model that the supply of the risky asset is stochastic.

(c) Explain why informed investors do not "arbitrage" the price of the financial asset p to equal $2/R$.

(d) Explain the procedure used to solve for the equilibrium market price of the financial asset.

(e) Explain in words how the following parameters affect the informational content of p: a, σ_ϵ^2, σ_x^2, and λ.

(f) Explain in words how the following parameters affect the equilibrium fraction of informed investors, λ: a, σ_θ^2, σ_x^2, and c.

(g) Discuss the differences and correspondences between this asset pricing model and the CAPM.

3. ADVERSE SELECTION AND ASSET PRICING

The role of information in asset pricing is quite different from that of risk. While information clearly does have an impact on the amount of risk that an individual faces, it has another more significant impact on asset pricing. Information obviously affects the perceived mean of the returns distribution, but, more pertinently, it might impart an *adverse selection* bias against an individual who possesses less information. Even, under risk neutrality, therefore, an uninformed investor may require a return premium to be willing to invest in a particular asset; the reason is that informed investors have a tendency to sell those assets on which they received negative information. The unconditionally expected mean return from the uninformed investor's perspective, even if unbiased in the absence of informed investors, is then below the mean return required by uninformed investors as it does not take account of the adverse selection bias. The adverse selection bias against uninformed investors is one prime explanation for why initial public offerings are frequently underpriced: uninformed investors receive their full allocation from their investment banker if the IPO is deemed undesirable by informed investors and are likely to rationed when the IPO is deemed attractive by informed investors. On this see, for instance, Beatty and Ritter (1984), Rock (1984) and Balvers et al. (1992).

Models with risk neutral investors or without systematic risk (in the traditional sense) may in principle explain cross-sectional differences in returns based on adverse selection biases. While there is an extensive theoretical literature dealing with asset pricing under asymmetric information [see for instance Wang (1993) for an interesting model], there is very little work done in terms of testing asymmetric information asset pricing models. One reason may be the difficulty in finding proxies to represent adverse selection risk. An approach to get at asymmetric information issues is by looking at SEC information about insider trading. The work of Seyhun (1998) provides evidence of abnormal returns obtainable by following the example of traders with inside information, where the extent of insider trading activity is observable from SEC publicly available documentation.

4. CROSS-SECTIONAL ASSET PRICING UNDER ASYMMETRIC INFORMATION

(a) Introduction

Little work exists on extending a model like that of Grossman and Stiglitz to an environment with multiple risky assets. The graduate texts on asset pricing by Cochrane (2001) and Campbell, Lo, and MacKinlay (1997) hardly pay any attention to how one may explain cross-sectional differences in expected return based on asymmetric information. This is however potentially a very important area of research. For instance, the "small firm" effect may be due to the fact that smaller firms are more closely held and may therefore impose, through insider trading, an adverse selection bias against uninformed/non-insider investors. Protecting small, uninformed investors is in fact a main reason for the existence of the SEC. Furthermore, firms in industries that are easier to audit may have different required returns as the degree of information content embodied in their market prices differs.

In the following we adapt the Grossman and Stiglitz model to allow multiple risky assets and then consider the model's implications for cross-sectional asset pricing. We make the same assumptions as in the original model with the following changes: (a) there are n risky assets; (b) the decision to become informed is ignored; (c) informed investors are informed about the θ_i of all assets. The latter is not realistic but simplifies analysis significantly and, certainly from the perspective of uninformed investors, it doesn't really matter who is informed but rather how much the informed as a group will purchase under different conditions.

Indicating vectors and matrices in bold we have the decision problem for each type of investor ($k = I, U$):

$$(1) \qquad \underset{\boldsymbol{X_k}}{\text{Max}} \quad E_k(w_k) - (a/2)\,\sigma^2_{w_k} \,.$$

$$(2) \qquad E_k(w_k) = R(\bar{w}_k - \boldsymbol{p}'\boldsymbol{X_k}) + E_k(\boldsymbol{u})'\,\boldsymbol{X_k} \,,$$

$$(3) \qquad \boldsymbol{u} = \theta + \boldsymbol{\varepsilon}.$$

Optimal portfolio choice by both types of investors yields:

$$(4) \qquad \boldsymbol{X_I^*} = \boldsymbol{\Sigma_\varepsilon^{-1}}(\theta - \boldsymbol{p}R)/a \,,$$

$$(5) \qquad \boldsymbol{X_U^*} = (\boldsymbol{\Sigma_\varepsilon} + \boldsymbol{\Sigma_{\theta|p}})^{-1}[\boldsymbol{E}(\theta|\boldsymbol{p}) - \boldsymbol{p}R]/a,$$

where Σ_{ε} and $\Sigma_{\theta|\boldsymbol{p}}$ are the covariance matrix for the $\boldsymbol{\varepsilon}$ and the conditional covariance matrix for the θ conditional on the available price information $\boldsymbol{p}$.

Market equilibrium for all risky asset markets (we can ignore the bond market from Walras's Law) is stated as:

$$(6) \qquad \lambda \boldsymbol{X}_{\boldsymbol{I}}^{*} + (1-\lambda)\boldsymbol{X}_{\boldsymbol{U}}^{*} = \boldsymbol{x}.$$

We guess the following solution for $\boldsymbol{p}$:

$$(7) \qquad \boldsymbol{p} = \boldsymbol{A}(\theta - \boldsymbol{Z}\boldsymbol{x}) + \boldsymbol{b},$$

where $\boldsymbol{A}, \boldsymbol{Z},$ are symmetric constant $n \times n$ matrices to be verified and $\boldsymbol{b}$ is a constant $n \times 1$ vector.

The relevant components needed to verify the solution for $\boldsymbol{p}$ in equation (7) from equation (6) are given as follows:

$$(7) \qquad \boldsymbol{E}(\theta\,|\,\boldsymbol{p}) = \bar{\theta} + \Sigma_{\theta\boldsymbol{p}}\Sigma_{\boldsymbol{p}}^{-1}(\boldsymbol{p} - \bar{\boldsymbol{p}}),$$

$$(8) \qquad \Sigma_{\theta\boldsymbol{p}} = \Sigma_{\theta}\boldsymbol{A}$$

$$(9) \qquad \Sigma_{\boldsymbol{p}} = \boldsymbol{A}(\Sigma_{\theta} + \boldsymbol{Z}\Sigma_{\boldsymbol{x}}\boldsymbol{Z})\boldsymbol{A}$$

$$(10) \qquad \Sigma_{\theta|\boldsymbol{p}} = \Sigma_{\theta} - \Sigma_{\theta\boldsymbol{p}}\Sigma_{\boldsymbol{p}}^{-1}\Sigma_{\boldsymbol{p}\theta}\,[\, = \Sigma_{\theta}\Sigma_{\boldsymbol{v}}(\Sigma_{\theta} + \Sigma_{\boldsymbol{v}})^{-1}\,].$$

The assumed solution in equation (7) can now be verified (tediously), with:

$$(11) \qquad \boldsymbol{A} = (\Sigma_{\boldsymbol{v}}^{-1} + \Sigma_{\theta}^{-1} + \lambda\Sigma_{\varepsilon}^{-1})^{-1}(\Sigma_{\boldsymbol{v}}^{-1} + \lambda\Sigma_{\theta}^{-1} + \lambda\Sigma_{\varepsilon}^{-1})/R,$$

$$(12) \qquad \boldsymbol{b} = (1-\lambda)(\Sigma_{\boldsymbol{v}}^{-1} + \Sigma_{\theta}^{-1} + \lambda\Sigma_{\varepsilon}^{-1})^{-1}(\Sigma_{\boldsymbol{v}}^{-1}\boldsymbol{Z}\bar{\boldsymbol{x}} + \Sigma_{\theta}^{-1}\bar{\theta})/R,$$

$$(13) \qquad \boldsymbol{Z} = (a/\lambda)\Sigma_{\varepsilon}^{-1},$$

$$(14) \qquad \Sigma_{\boldsymbol{v}} \equiv \boldsymbol{Z}\Sigma_{\boldsymbol{x}}\boldsymbol{Z}.$$

Rewrite equation (7) for easier interpretation into expected payoff, risk adjustment, and random components:

$$(15) \qquad \boldsymbol{p} = (\bar{\theta}/R) - (a/R)(\Sigma_{\varepsilon} + \Sigma_{\theta} - \boldsymbol{R}\boldsymbol{A}\Sigma_{\theta})\bar{\boldsymbol{x}} + \boldsymbol{A}[\theta - \bar{\theta} - \boldsymbol{Z}(\boldsymbol{x} - \bar{\boldsymbol{x}})].$$

The first term indicates the expected present value of the payoff for each asset; the third term indicates the zero-expected value error term. The middle term in equation (15) indicates the risk adjustment. If there were no asymmetric information, $\lambda = 0$, then the risk adjustment would be the standard $\Sigma_{\varepsilon} + \Sigma_{\theta}$ directly related to the payoff risk based on payoffs as given in equation (3). The risk adjustment is proportionate to the covariance matrix $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} + \boldsymbol{\Sigma}_{\boldsymbol{\theta}} - \boldsymbol{R}\,\boldsymbol{A}\,\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$ which is the covariance matrix of $\boldsymbol{u} - R\boldsymbol{p}$, the vector of realized payoffs net of the opportunity cost of investment.

The introduction of asymmetric information does not add to risk, as one might expect given an adverse selection bias imposed on the uninformed. Rather the risk adjustment is lower due to the fact that informed investors learn the risk associated with θ and "remove" it from the market; in fact, they only remove part of this risk from the market since prices are not fully revealing. The part that is removed depends on how revealing prices are about θ, which, in turn, depends on the covariance between $\boldsymbol{p}$ and θ. This covariance is given as from equation (7) as $\boldsymbol{\Sigma}_{\boldsymbol{\theta p}} = \boldsymbol{A}\,\boldsymbol{\Sigma}_{\boldsymbol{\theta}}$, which is proportional to the additional term in the risk adjustment. Interestingly, the price variability based on the information incorporated in prices due to informed traders is a "good" variability and raises the average value of the assets. Note that it is straightforward to obtain the intuitive result that at the extreme where all investors are informed, $\lambda = 1$, the risk adjustment is simply equal to $\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}}$.

(b) Cross-sectional Asset Pricing

Equation (15) is not directly useful in explaining differences in expected returns across assets. It is however possible to construct a two-beta CAPM based on the above model. We will consider all expectations from the unconditional perspective of the empirical researcher collecting average returns information. Rewrite equation (15), using equation (3), as:

$$(16)\qquad \boldsymbol{u} - R\boldsymbol{p} = a\,(\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} + \boldsymbol{\Sigma}_{\boldsymbol{\theta}} - \boldsymbol{R}\,\boldsymbol{A}\,\boldsymbol{\Sigma}_{\boldsymbol{\theta}})\,\bar{\boldsymbol{x}} - \boldsymbol{R}\boldsymbol{A}[\theta - \bar{\theta} - \boldsymbol{Z}(\boldsymbol{x} - \bar{\boldsymbol{x}})] + \theta - \bar{\theta} + \boldsymbol{\varepsilon}.$$

"Dividing" both sides of equation (16) by the vector $\boldsymbol{p}$ provides an expression for excess returns. This is fine from the perspective of uninformed investors who perceive prices, but from an unconditional perspective, without knowledge of realized prices, these returns are not normally distributed (the ratio of two normally distributed variables is not normal and has an ill-defined distribution with moments that do not exist). Hence we will continue to work with excess payoffs rather than excess returns.

Interpreting the variance terms as covariances we obtain:

$$(17)\qquad Cov(\boldsymbol{u}, w_m) = (\boldsymbol{\Sigma}_{\boldsymbol{\varepsilon}} + \boldsymbol{\Sigma}_{\boldsymbol{\theta}})\,\bar{\boldsymbol{x}}\;, \qquad Cov(\boldsymbol{p}, \theta_m) = \boldsymbol{\Sigma}_{\boldsymbol{p\theta}}\,\bar{\boldsymbol{x}} = \boldsymbol{A}\,\boldsymbol{\Sigma}_{\boldsymbol{\theta}}\,\bar{\boldsymbol{x}}.$$

Note that you must use Stein's generalized lemma (see Appendix C) to derive equation (17).

The expected payoff vector becomes:

$$(18)\qquad \bar{\theta} - R\bar{\boldsymbol{p}} = a\,Cov(\boldsymbol{u}, w_m) - a\,R\,Cov(\boldsymbol{p}, \theta_m).$$

The expected net payoffs in quation (18) include the standard CAPM compensation for systematic covariance risk related to the co-movement of the payoff with market wealth, but also includes an additional covariance term related to asymmetric information. Since informed investors take on much of the "risk" due to the learnable component θ, assets which have much exposure to this component are overpriced based on the CAPM risk and in effect have risk that is less compensated in the market than would be true in the pure CAPM context. Accordingly, its expected payoff needs to be lower by a factor related to how the asset's price covaries with systematic risk related to the learnable component.

To derive a two-beta CAPM form, take the *i*-th element from the expected return vector in equation (18):

$$\bar{\theta}_i - R\,\bar{p}_i = a\,Cov(u_i, w_m) - a\,R\,Cov(p_i, \theta_m). \tag{19}$$

Aggregate equation (19) over all assets *i* weighted by their expected available supplies $\bar{x}_i$; this yields the market expected excess payoff:

$$\bar{\theta}_m - R\,\bar{p}_m = a\,Var(w_m) - a\,R\,Cov(p_m, \theta_m)\ . \tag{20}$$

Consider now that an asset *0* exists which has zero uncertainty about the learnable component (so that $u_0 = \bar{\theta}_0 + \varepsilon_0$). Its excess expected payoff would be:

$$\bar{\theta}_0 - R\,\bar{p}_0 = a\,Cov(u_0, w_m) - a\,R\,Cov(p_0, \theta_m). \tag{21}$$

One might expect that the last term in equation (21) is zero, but this is not guaranteed and is not important for obtaining the CAPM expression. Using equations (20) and (21) to solve for expressions in *a* and *aR* and substituting into equations (19) gives:

$$\bar{\theta}_i - R\,\bar{p}_i = \beta_{p1}(\bar{\theta}_m - R\,\bar{p}_m) + \beta_{p2}(\bar{\theta}_0 - R\,\bar{p}_0). \tag{22}$$

A more applicable form would arise if we divided in equations (19)-(21) by $\bar{p}_j$ ($j = i, m, 0$) and also used the fact that $w_m = \bar{w}_m R_m$. Then:

$$\bar{R}_i - R = \beta_1(\bar{R}_m - R) + \beta_2(\bar{R}_0 - R), \tag{23}$$

where expected returns are obtained by dividing (somewhat inappropriately) by average price rather than actual price. Note that the betas obtained here do not have the form of multiple regression coefficient, but that, of course, there should theoretically by equivalence between the theoretical and the multiple regression coefficients.

Empirically, the challenge is to find an asset or portfolio *0* which has no or small learnable component. One might consider firms which do not attract financial analysts, maybe "boring" firms such as utilities; or one might look at firms for which most of the variation in price occurs at the time of public announcements. Alternatively, one might look at firms for which little insider trading activity is reported; an issue that we discuss next..

(c) Applications and exercises

1. Derive the Asymmetric Information CAPM under the assumption that a risk free asset exists but does not have

a predetermined return.

2. Theoretically analyze the typical sign of the beta coefficient on both factors in the asymmetric information CAPM.

3. Explain why there is no “adverse selection” risk premium in the asymmetric information version of the CAPM.

4. Obtain the CAPM expression from the conditional perspective of the uninformed investors.

5. What portfolio advice would you give to uninformed investors?

5. INSIDER TRADING

The most typical case in which information is asymmetrically distributed is when “insiders” are involved. In general terms, insiders are those who have first-hand knowledge about the value of the asset they trade. In more specific terms, insider trading refers to the transactions of the officers, directors, and large shareholders of a firm when they trade the stock of their firm. We consider several issues regarding insider trading: its regulation, whether insider gain from their trades, whether outsiders at times gain by following the example of insiders, and, if so, under which specific conditions outsiders benefit by mimicking insiders. These questions are answered nicely by Seyhun (1998). The following discussing is based on this work.

Regulations covering insider trading activities

First consider that most insider trading is in fact legal but must be reported. This means that reliable data are available to study and mimic the actions of insiders. The legal definition takes insiders to be officers or directors of a company, or any individual or corporation owning more than 10% of a particular equity class in the company. To decide which officers are covered, the test is whether the officer has decision authority that affects the entire organization. Insider trading regulations require insiders to disclose all their stock market transactions in their own firm to the Securities and Exchange Commission (SEC) on a “timely” basis, meaning within the first ten days of the calendar month following the month in which the transaction occurred. Failure to do so may result in fines of up to $2.5 million and ten years of jail time.

What insider trading then is illegal? Insider trading restrictions are covered by Section 10 of the Securities and Exchange Act of 1934: trading by individuals based on *material, nonpublic* information is illegal. The term “material, nonpublic information” is, however, not defined specifically either by the SEC or U.S. courts. In practice, therefore, most insider transactions are considered to be legal. They can be profitable as well as legal, for instance, when insiders are better able to interpret public information. According to Section 16 of the Securities and Exchange Act of 1934, insiders are not allowed to benefit from buying and selling their stock within a half year. If profits are generated by such short-term trading, they need to be repaid to the corporation. This requirement makes it difficult for insiders to manipulate stock prices as at least two quarterly reports must appear before a buy or sell transaction can be reversed to lock in a profit. Section 16, further, prohibits insiders from short-selling their stock. As a result, insiders cannot benefit from deliberate mismanagement; they also cannot exploit negative information. Lastly, Section 16 requires the timely disclosure of insider trading, which forces public dissemination of insider trading activity and makes it easier to detect violations. These restrictions remove much of the incentives for insiders to try to manipulate their own stock, and better align the interests of the corporate officers and directors with those of the stockholders. Individual firms may have private trading restrictions that are much more severe, such as individuals not being allowed to trade in competitors' stock.

The federal insider trading laws were passed in response to the crash of 1929 and were intended to protect small investors. The idea is that they lower adverse selection risk to small investors by making it less likely that they end up trading with someone who has superior information. As a result, small savers could invest without fear, providing additional liquidity to the stock market and, in principle, benefitting insiders.

Current laws trade violations of insider-trading laws almost as seriously as violent crimes. Additionally, over time SEC enforcement efforts have increased. The SEC now initiates 40 to 45 insider trading cases per year, about 15 times as much as during the 1970s. The SEC is allowed to pay informants up to 10% of the profits of the illegal trades on which they report. While the list of insiders who must disclose trades is clearly defined, nobody is allowed to benefit from trading on insider information. This includes trading on a "tip" you receive from the spouse of a janitor working for the company. Insider trading, nevertheless, has not changed dramatically over time and is quite common. Seyhun (1998) indicates that in his data set over the period from January 1975 to December 1995, 65% to 80% of all publicly listed firms report at least one insider transaction per year. The unconditional probability of trading against an insider, however, is small: 2% of the trading volume in small firms and 0.5% of the trading volume in large firms is due to insider transactions.

With data, available from the SEC, of all insider transactions since 1975 Seyhun (1998) examines the profitability of insider transactions. He classifies a particular month as a "buy" ("sell") month for a particular stock if the net number of shared purchased by insiders is positive (negative). Stocks outperform the market by 4.5% in the year following a buy month; stocks underperform the market by 2.7% in the year following a sell month. These excess returns accumulate slowly over the course of the year. Hence, outsiders, by mimicking insiders could buy "buy" stocks and sell "sell" stocks and with a zero-investment portfolio could generate an excess return of 7.2% per dollar sold short.

By focusing solely on strong signals, even higher excess returns can be generated. If we exclude "passive" transactions–those that reverse a previous sale or purchase–or consider only signals that are non-conflicting, for instance buying in "buy" months with no disagreeing sales or "sell" months in the year prior, then total excess returns on a zero-investment portfolio may increase to as high as 20.0% per dollar sold short. Moreover, if we separate insiders into top executives, directors, officers, and large stockholders, the excess returns in response to top executives' transactions are highest and those in response to large stockholders' transactions lowest. Also, up to a certain maximum, large trades (by all groups but the large stockholders) have more information value. Insider trading in small firms is a lot more valuable. All these signals, in principle, can be used by outsiders to generate excess returns. Impressively, the profitability to insider transactions in the months preceding a "sell" ("buy") month is similarly positive. While outsiders cannot mimic these transactions, the evidence of excess returns in these cases provides additional confidence in the robustness of the results.

Seyhun (1998) also shows that *aggregate* insider trading provides information about *aggregate* stock market returns. Define aggregate insider trading as the net proportion of
firms exhibiting insider "buying", where "buying" is defined as 50% or more of the firms reporting net buying activity during the past 12 months. Then the (equal-weighted) market return was 28.7% in years following a "buying" year (seven such years in the sample), while the market return was only 12.9% in "selling" years (12 such years in the sample). Further, the aggregate signals may be combined with the firm-specific signals to generate even higher excess returns.

Next Seyhun investigates how the information content in insider trading correlates with the information in other variables known to forecast future returns. The results for dividend yield, earnings-price ratio, book-to-market ratio and mean-reversion potential are comparable. Each of these variables are high for "value" stocks and low for "growth" stocks, and each of these variables predicts future returns in the sense that value stocks generate significantly higher returns than growth stocks. Seyhun reaches three basic conclusions regarding these variables: (1) insiders tend to buy value stocks, i.e., stocks with high dividend yield, high earnings-price ratios, high book-to-market ratios and high mean-reversion potential, and tend to sell growth stocks; (2) the insider trading variables retain their forecasting power even when considered together with these additional variables; (3) the forecasting power of the four value/growth variables diminishes when the insider trading variable is also considered, and more so when the strongest insider trading signals are used. A similar conclusion is reached for earnings surprises: they forecast future returns but their effect is attenuated when insider trading signals are added.

While it is not surprising that inside traders are able to generate additional profits for themselves, what is surprising is that the excess returns are spread out over a full year (or possibly longer). Given that the mean reporting delay is 26 days, more than half the instances of insider trades are available to the public within a month of their occurrence. Outsiders thus can reap substantial excess returns by observing insider trading signals. This is a puzzle since it is likely that insiders trade based on information that is important to the expected value of the firm and not based on risk prospects. So, it appears that outsiders can obtain excess returns without needing to incur additional risk, just by mimicking insiders.