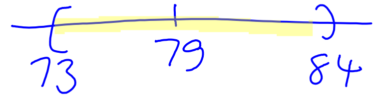
2013-11-12-EC01

Tuesday, November 12, 2013

6:02 PM

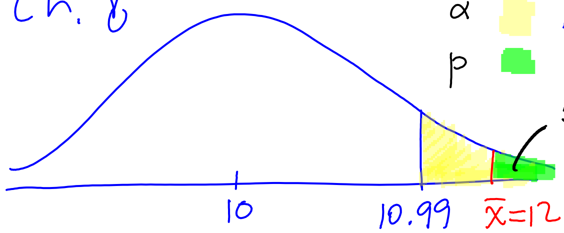
95% CI for exam μ

$n = 8$ $\bar{x} = 79$ $s = 7$



Back to Ch. 8

$\alpha = .05$



$H_0: \mu \le 10$

$H_a: \mu > 10$

$p < \alpha$: reject H_0

Ex. GMAT Scores (2005)

In 2005, $\mu = 630$, $\sigma = 45$

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/GMAT-05.xls

GMAT-2005
690
560
680
660
620
580
650
590
620

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	GMAT-2005
count	101
mean	630.50
minimum	490
maximum	720
range	230
population variance	1,919.56
population standard deviation	43.81

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Dean believes that in 2006

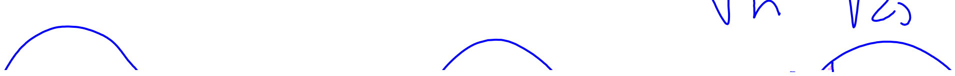
$H_0: \mu \ge 630$

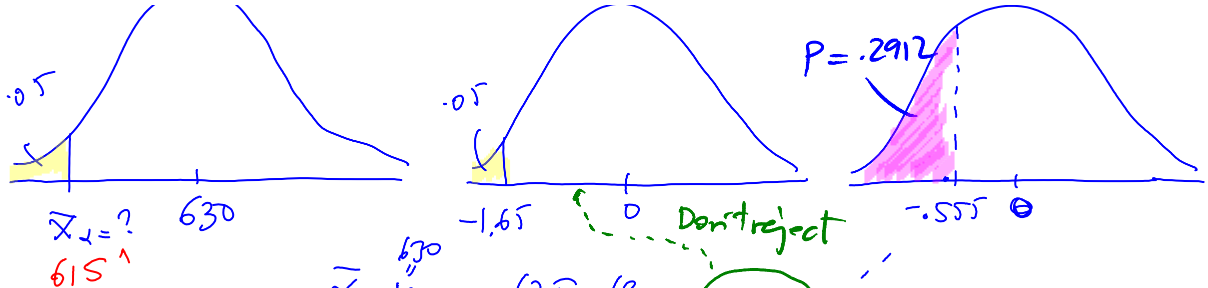
$H_a: \mu < 630$

Around May '06, $n = 25$, $\bar{x} = 625$ $(\sigma = 45)$

Reject H_0 at $\alpha = .05$?

$$\frac{\sigma}{\sqrt{n}} = \frac{45}{\sqrt{25}} = 9$$





$$z = \frac{\bar{x} - \overset{630}{\mu_0}}{\sigma/\sqrt{n}} = \frac{625-630}{9} = -.555$$

What's $\bar{x}_\alpha$?

$$-1.65 = \frac{\bar{x}_\alpha - 630}{9} \Rightarrow \boxed{\bar{x}_\alpha = 615}$$

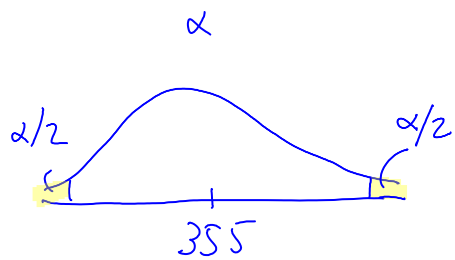
Ex. Bottling comp

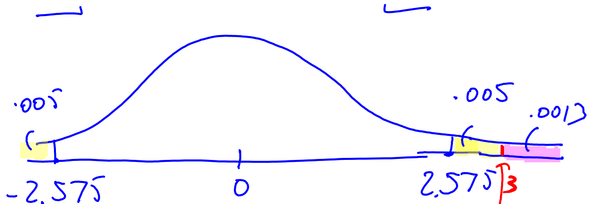
pb. 8-11, 8-45
p. 259, 274

Ideal amt 355 mL

H_0: $\mu = 355$

H_a: $\mu \neq 355$





$\alpha = .01$, $n = 36$, $\sigma = .1$

$$z = \frac{\bar{x} - 355}{0.1/\sqrt{36}}$$

a) $\bar{x} = 355.05$, $\boxed{z = 3}$ Reject

$p = 2(.0013) = .0026 < .01$ reject

b) t-test about (σ unknown)

If σ not known, use s from sample

If σ not known, use s from sample

$$t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}} \quad : df = n-1$$

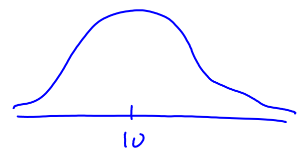
(Similar to CI material)

Ex. Tar content

$H_0: \mu \le 10$ " μ_0

$H_a: \mu > 10$

σ unknown



$\alpha = .05, \quad n = 10$

9
11
10.5
12
9.5
10
10.5
9
8
13

$n = 10$

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10.0000	hypothesized value		
10.2500	mean Data		
1.4954	std. dev.		
0.4729	std. error		
10	n		
9	df		
0.53	t		
.3049	p-value (one-tailed, upper)		

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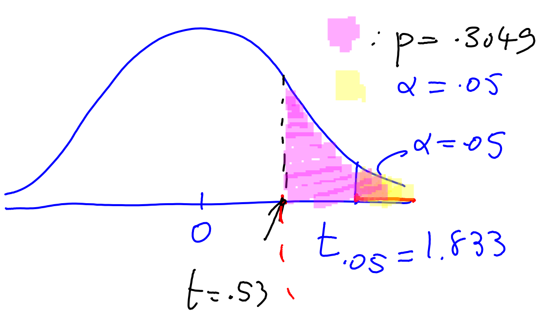
$\bar{x} = 10.25, \quad s = 1.4954$

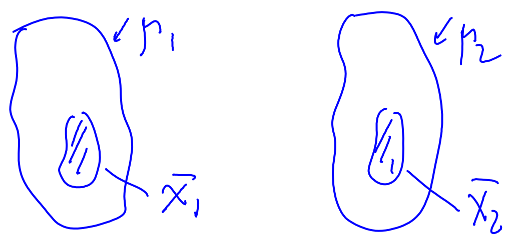
$\frac{s}{\sqrt{n}} = .4729$

$$t = \frac{10.25 - 10}{.4729} = \boxed{.53}$$



Don't reject

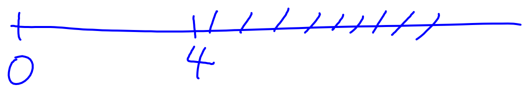
Ch. 9 Statistical inference based on two samples (exclude 9.2 → 9.5)



HT. $H_0: \mu_1 - \mu_2 \geq D_0$ $\left(\begin{matrix} = D_0 \\ \leq D_0 \end{matrix}\right)$

$H_a:$ ~~~~

Ex. Atkins vs. Conv.



A C

$\mu_1 - \mu_2 \geq 4$

So, $H_0: \mu_1 - \mu_2 \geq 4$

$H_a: \mu_1 - \mu_2 < 4$

[9.1] z-test for $\mu_1 - \mu_2$ (σ_1^2, σ_2^2 known)

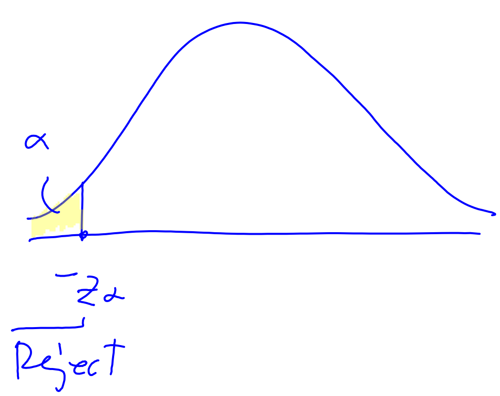
Recall

One pop

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$$

Two pop's

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}}$$ } ch.7



Ex. Atkins vs. Conventional

Ex. Atkins vs. Conventional

$$H_0: \mu_1 - \mu_2 \geq 4 \qquad (D_0 = 4)$$

$$H_a: \mu_1 - \mu_2 < 4$$

	Initial	6-month	Loss at			Initial	6-month	Loss at
Diet	Weight (lbs)	Weight	6 Months (Atk)		Diet	Weight (lbs)	Weight	6 Months (Con)
Atkins	310	292.7	17.3	1	Conventional	256	259.5	-3.5
Atkins	309	275.1	33.9	2	Conventional	198	186.3	11.7
Atkins	257	217.7	39.3	3	Conventional	311	299.9	11.1
Atkins	227	221.1	5.9	4	Conventional	246	231.6	14.4
Atkins	231	204.5	26.5	5	Conventional	170	182.2	-12.2

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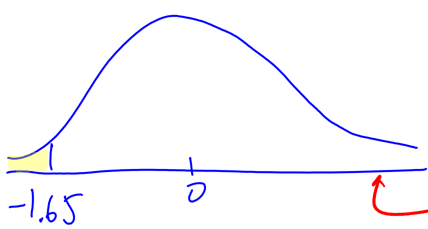
A	C
$n_1 = 33$	$n_2 = 30$
$\bar{x}_1 = 15.42$	$\bar{x}_2 = 7.00$
$\sigma_1 = 8$	$\sigma_2 = 6$

$$D_0 = 4$$

$$z = \frac{(\bar{x}_1 - \bar{x}_2) - D_0}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(8.42) - 4}{\sqrt{\frac{8^2}{33} + \frac{6^2}{30}}} = \boxed{2.49}$$

$\alpha = .05$

$z_\alpha = 1.65$



Don't reject H_0

Exclude [9.2] H_a: $\mu_1 - \mu_2 \neq D_0$

[9.3 & 9.4] t-test $\mu_1 - \mu_2$

[9.5] proportion

[9.6] F-test for equality of variances, i.e.,

$\sigma_1^2 = \sigma_2^2$

(Was needed 7.6 $\uparrow$ CI)

Ex. Atkins data

	6 Months (Atk)
count	33
mean	15.424
sample variance	206.528
sample standard deviation	14.371
minimum	-17.3
maximum	47
range	64.3

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$s_1 = 14.37$

$s_1^2 = 206.50$

	6 Months (Con)
count	30
mean	7.007
sample variance	152.788
sample standard deviation	12.361
minimum	-12.9
maximum	36.5
range	49.4

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$s_2 = 12.36$

$s_2^2 = 152.78$

We have $s_1^2 > s_2^2$, but is evidence strong enough to reject $\sigma_1^2 = \sigma_2^2$

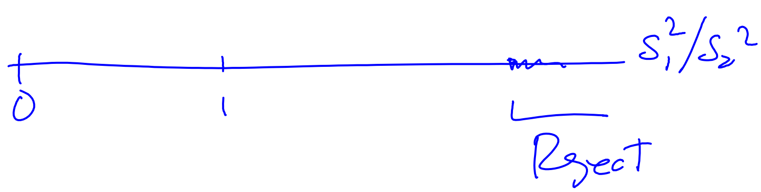
$H_0: \sigma_1^2 = \sigma_2^2$

$H_a: \sigma_1^2 > \sigma_2^2$

$H_0: \dfrac{\sigma_1^2}{\sigma_2^2} = 1$

$H_a: \dfrac{\sigma_1^2}{\sigma_2^2} > 1$

Intuition



Result (Fisher).

If H_0 is true, then

$\dfrac{s_1^2}{\sigma_1^2}$ is F-distributed with

s_2^2

df_1: numerator (n_1-1)

df_2: denominator (n_2-1)

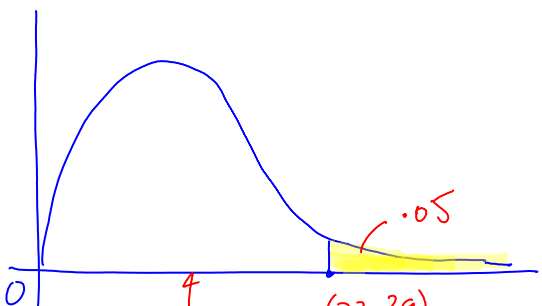
Atkins	Conventional
$n_1=33$	$n_2=30$
$s_1^2=206.5$	$s_2^2=152.78$
$df_1=32$	$df_2=29$

Test stat

$$F=\frac{s_1^2}{s_2^2}=\frac{206.5}{152.78}=\boxed{1.35}$$

$n_1-1=32$
$n_2-1=29$
$\alpha=.05$

$\alpha=.05$

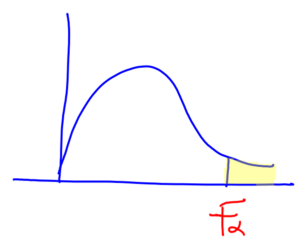


Don't reject H_0

$F_{.05}^{(32,29)} \simeq 1.85$

Remark If $s_2^2 > s_1^2$

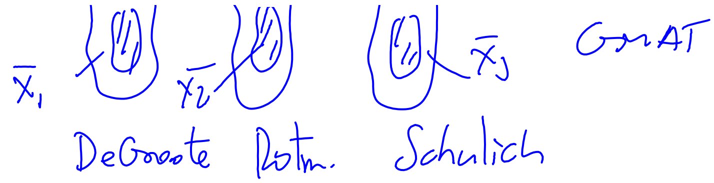
$$\left.\begin{array}{l} H_0:\ \sigma_2^2=\sigma_1^2 \\ H_a:\ \sigma_2^2>\sigma_1^2 \end{array}\right\} \quad F=\frac{s_2^2}{s_1^2}$$



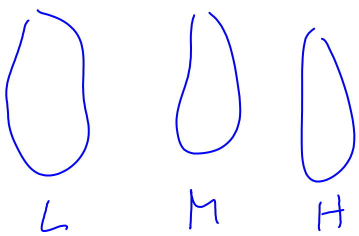
See text

Ch.10 Experimental design & Analysis of Variance (ANOVA)

Ex.

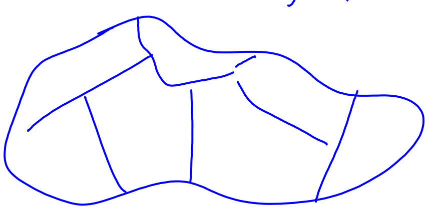


Ex. Fertilizer



a) Experimental design

Ex. Fertilizer level $\longrightarrow$ yield
L, M, H



Factor (ind't var) $\xrightarrow{\text{influence}}$ dep't var
e.g. fertilizer   e.g. yield
moisture
⋮

levels of factor(s): treatments $\longrightarrow$ yield level
L, M, H

Ex. Experimental farm

1 L	2 M	3 L	4 H	5 L	6 M
7 L	8 H	9 M	10 H	11 H	12 L
13 M	14 H	15 L	16 H	17 M	18 M

Completely randomized one-way experimental design (single factor with $k=3$ levels)

L: 3, 7, 1, 12 . .

M: . .

H:

b) One-way Analysis of Variance (ANOVA)

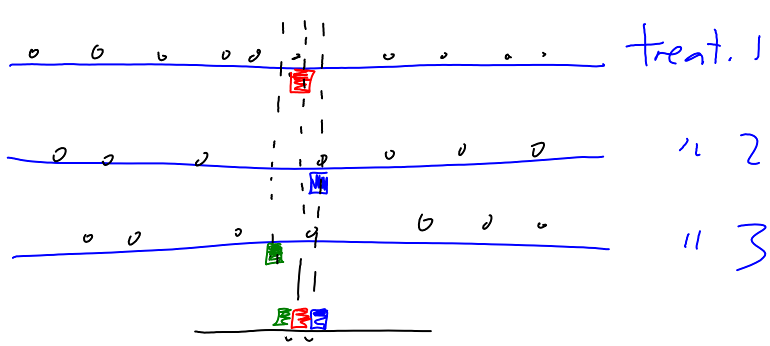
Ex. Three popns

$H_0: \mu_1 = \mu_2 = \mu_3$

Ha: at least two of μ_1, μ_2, μ_3 are different

either
$\mu_1 \neq \mu_2$ or
$\mu_1 \neq \mu_3$ or
$\mu_2 \neq \mu_3$ or
$\mu_1 \neq \mu_2 \neq \mu_3$

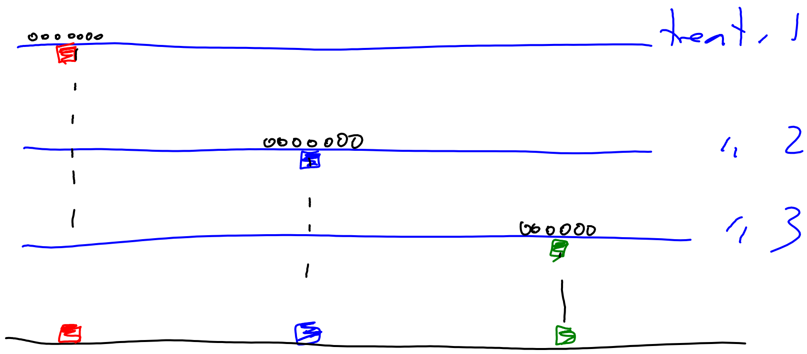
Case 1



· Between treatment variability is not large compared to within treatment variability large compared to within treatment variability

H_0 : don't reject!

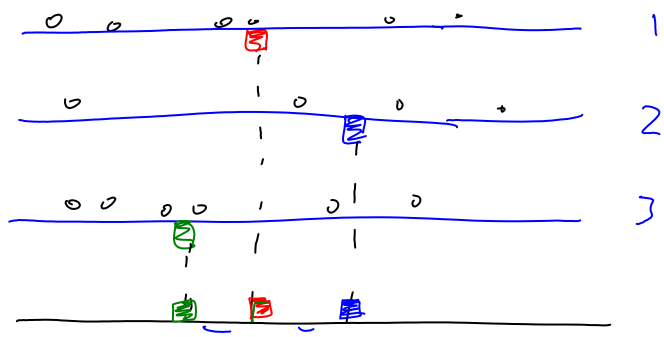
Case 2



Large variability between treatments vs. within treatment

Reject H_0 !

Case 3



Not too clear! Use ANOVA

p treatments

$$H_0: \mu_1 = \mu_2 = \dots = \mu_p$$

H_a: at least two differ

Assumptions

① Pop. variances are equal

② " normal

③ samples are independent

(3) samples are independent