2013-11-07-C02

Thursday, November 07, 2013
9:59 AM

NO CELL PHONES IN CLASS!

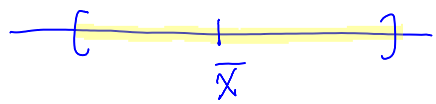
e) [7.5] CI for $\mu_1 - \mu_2$: Ind't Samples σ_1^2, σ_2^2

Recall [7.1] Single pop'n

$\bar{X}$: sample mean $\begin{cases} \mu_{\bar{X}} = \mu \\ \sigma_{\bar{X}} = \dfrac{\sigma}{\sqrt{n}} \end{cases} \Rightarrow \sigma_{\bar{X}}^2 = \dfrac{\sigma^2}{n}$

$100(1-\alpha)\%$ CI for μ,

$\left[\bar{x} \mp z_{\alpha/2} \dfrac{\sigma}{\sqrt{n}}\right]$:

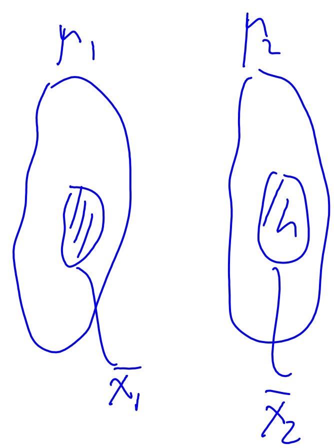


Two popn's

$\bar{X}_1 - \bar{X}_2$: diff in sampls

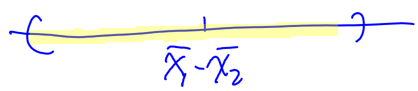
$\mu_{\bar{X}_1 - \bar{X}_2} = E(\bar{X}_1 - \bar{X}_2) = \mu_1 - \mu_2$

$\sigma^2_{\bar{X}_1 - \bar{X}_2} = Var(\bar{X}_1 - \bar{X}_2) = \dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}$



$100(1-\alpha)\%$ CI for $\mu_1 - \mu_2$

$\left((\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2} \sqrt{\dfrac{\sigma_1^2}{n_1} + \dfrac{\sigma_2^2}{n_2}}\right]$



Ex. Atkins vs. conventional diet

	Diet	Initial Weight (lbs)	6-month Weight	Loss at 6 Months (Atk)		Diet	Initial Weight (lbs)	6-month Weight	Loss at 6 Months (Con)
1	Atkins	310	292.7	17.3	1	Conventional	256	259.5	-3.5
2	Atkins	309	275.1	33.9	2	Conventional	198	186.3	11.7

3	Atkins	257	217.7	39.3	3	Conventional	311	299.9	11.1
4	Atkins	227	221.1	5.9	4	Conventional	246	231.6	14.4
5	Atkins	231	204.5	26.5	5	Conventional	170	182.2	-12.2
6	Atkins	195	148	47	6	Conventional	244	251.2	-7.2
7	Atkins	190	179.7	10.3	7	Conventional	179	180	-1
8	Atkins	164	131.1	32.9	8	Conventional	204	202.1	1.9
9	Atkins	190	162.7	27.3	9	Conventional	261	265.5	-4.5
10	Atkins	140	125.2	14.8	10	Conventional	271	260.3	10.7
11	Atkins	251	240.8	10.2	11	Conventional	265	263	2

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	6 Months (Atk)
count	33
mean	15.424
sample variance	206.528
sample standard deviation	14.371
minimum	-17.3
maximum	47
range	64.3

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$n_1 = 33$

$\bar{x}_1 = 15.42$

$\sigma_1 = 8$

available

$\sigma_2 = 6$

	6 Months (Con)
count	30
mean	7.007
sample variance	152.788
sample standard deviation	12.361
minimum	-12.9
maximum	36.5
range	49.4

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$n_2 = 30$

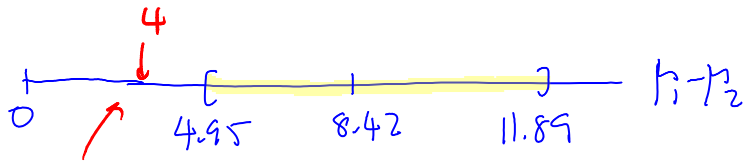
$\bar{x}_2 = 7.00$

95%

$z_{.025} = 1.96$

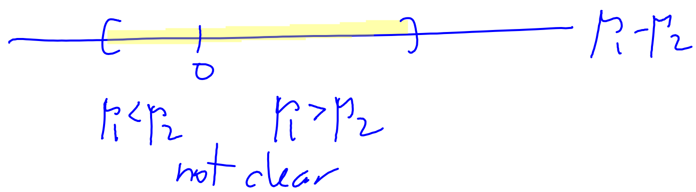
$$\left[(\bar{x}_1 - \bar{x}_2) \pm z_{\alpha/2}\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}\right]$$

$$= \left[8.42 \mp 1.96\sqrt{\frac{8^2}{33} + \frac{6^2}{30}}\right] = [4.95, 11.89]$$



atkins (more effective than conventional)

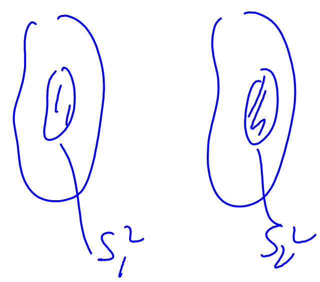
If we had



f) [7.6] CI for $\mu_1 - \mu_2$: Ind't samples
σ_1^2, σ_2^2 unknown
but equal; $\sigma_1^2 = \sigma_2^2 = \sigma^2$

Pooled estimate for σ^2

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2}$$



CI: $\left[(\bar{x}_1 - \bar{x}_2) \mp t_{\alpha/2}\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}\right]$, $df = n_1+n_2-2$

Ex. Atkins vs Conv

A	C
$n_1 = 33$	$n_2 = 30$
$\bar{x}_1 = 15.42$	$\bar{x}_2 = 7.00$
$s_1 = 14.37$	$s_2 = 12.36$

So,
$$s_p^2 = \frac{(32)(14.37)^2 + (29)(12.36)^2}{33+30-2} = 73.72$$

$df = 33+30-2 = 61 \rightarrow t_{\alpha/2} = t_{.025} = 2.00$

95% CI: $\left[8.42 \pm 2.00\sqrt{73.72\left(\frac{1}{33}+\frac{1}{30}\right)}\right]$
$= [4.08, 12.75]$

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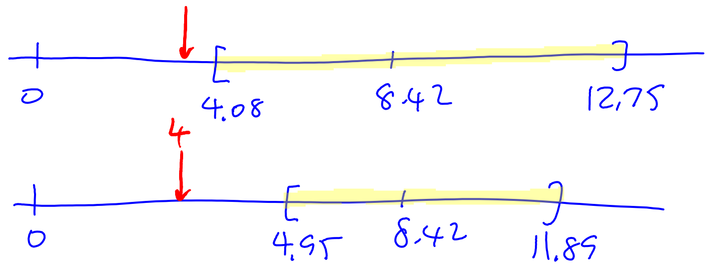


σ_1^2, σ_2^2 known

Ch. 8. Hypothesis testing

Ex. "The Lady Tasting Tea"
Arief " Coke & Pepsi

	1	2	3	4	5	6	7	8	# Correct
	C	C	C	C	P	P	P	P	
$\binom{8}{4} = 70$	C	C	C	C	P	P	P	P	4+4=8
	P	C	C	C	P	C	P	P	3+3=6
	P	P	P	P	C	C	C	C	0+0=0
			⋮			⋮	⋮		

$$\Pr(\text{identify all correctly}) = \frac{1}{70} = 1.4\%$$

Arief got all right!
$\Rightarrow$ reject my hypothesis that he was guessing

a) Null & alternative hypotheses

z-tests about μ (σ known)

Ex. legal process

— Person charged

H_0: person innocent
H_a: " guilty

is innocent until proven guilty

Actions ① Reject H_0
② Accept (don't reject) H_0

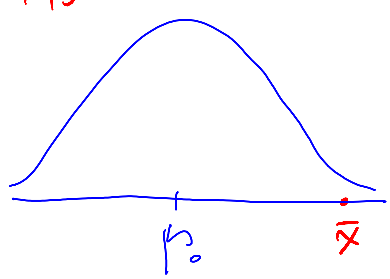
		innocent H_0 true	guilty H_0 false
Decisions	(find guilty) Reject H_0	Type I error	Correct
	(" innocent) Accept H_0	Correct	Type II error

$$\alpha = Pr(\text{Type I error}) = Pr(\text{Reject } H_0 \mid H_0 \text{ true})$$

$$\beta = Pr(\text{Type II} \ \text{"}) = Pr(\text{Accept } H_0 \mid H_0 \text{ false})$$

Intuition for rejecting H_0

Suppose $H_0: \mu \le \mu_0$
$H_a: \mu > \mu_0$



Ex. Tar content of cigarettes

- $H_0: \mu \le 10$
  $H_a: \mu > 10$ } One-sided

$n = 25$, $\sigma = 3$, $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{3}{\sqrt{25}} = .6$

- $\alpha = Pr(\text{Type I error}) = Pr(\text{reject } H_0 \mid H_0 \text{ true})$

- $\alpha = Pr(\text{Type I error}) = Pr(\text{reject } H_0 \mid H_0 \text{ true})$

  $\alpha = .05$

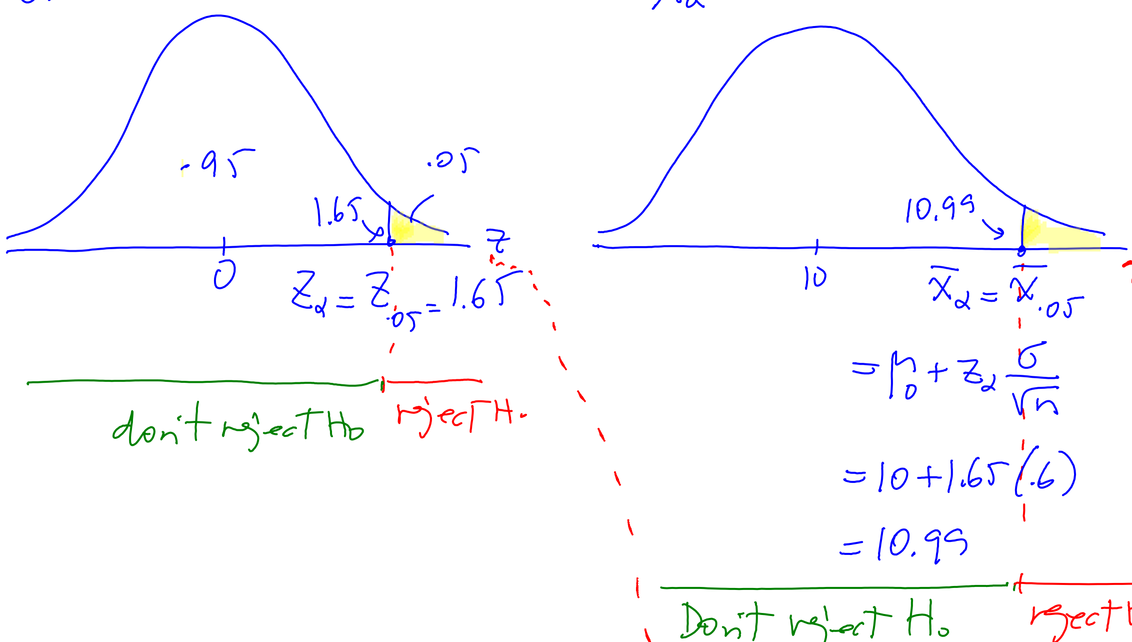
- Test statistic

  - Replace $H_0: \mu \le 10$ by $H_0: \mu = 10$

  - Select sample and find $\boxed{\bar{x}}$
  - Or use $\boxed{z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}}$

- Find rejection point z_α & region for given α

  Or " " " $\bar{x}_\alpha$ " " " " "



$z_\alpha = z_{.05} = 1.65$

$\bar{x}_\alpha = \bar{x}_{.05}$

$= \mu_0 + z_\alpha \dfrac{\sigma}{\sqrt{n}}$

$= 10 + 1.65(.6)$

$= 10.99$

- Collect data and find $\bar{x}$ and z

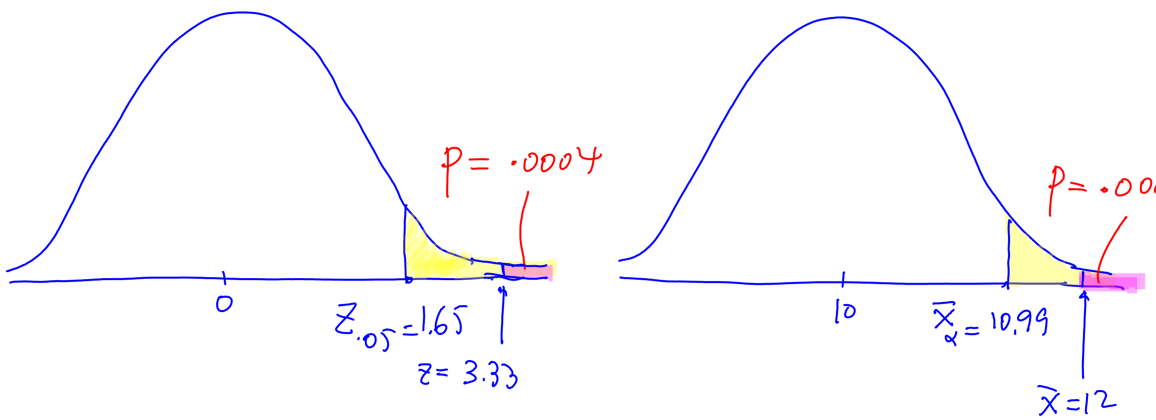
  $n = 25$, $\boxed{\bar{x} = 12}$

  reject H_0

  reject H_0

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{12-10}{3/\sqrt{25}} = \boxed{3.33}$$

The p-value for testing hypothesis



Evidence to reject H_0