2013-10-10-C02

Thursday, October 10, 2013
11:27 AM

Binomial

$$p(x) = \binom{n}{x} p^x (1-p)^{n-x}, \quad x = 0, 1, \ldots, n$$

Ex. New drug

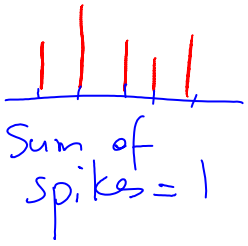
http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Binomial-NewDrug.pdf

$$p(0) = \binom{8}{0} (.7)^0 (.3)^8$$

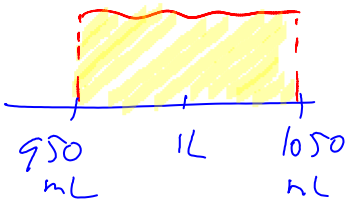
Ch. 5 Continuous random variables

a) Cont. distributions

Discrete



Continuous

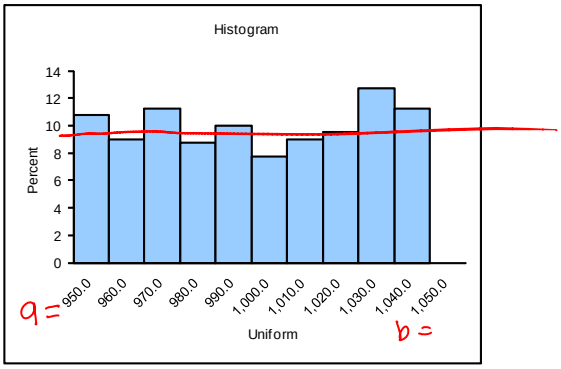


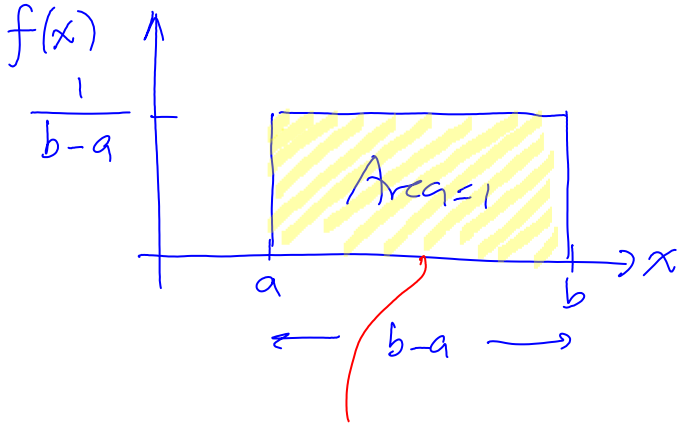
Area = 1

http://www.youtube.com/watch?v=dn1mIZS6I6o&feature=related

b) Uniform distribution

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/AppleJuice.xls



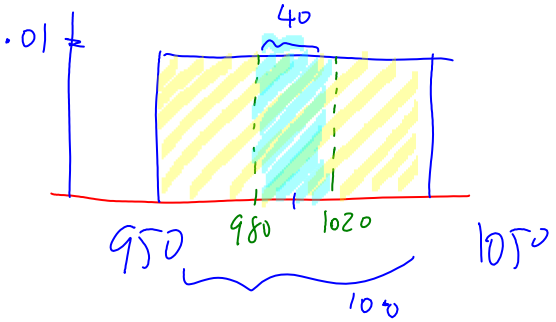


Mean $\mu_X = \frac{1}{2}(a+b)$

S.d. $\sigma_X = \frac{b-a}{\sqrt{12}}$

Ex. Apple juice

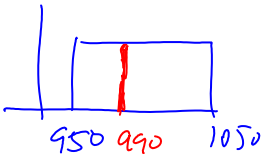
$\pm 2\%$ deviation is OK



$Pr(980 \le X \le 1020) = 40 \times .01 = .4$

24

$Pr(X = 990) = 0$

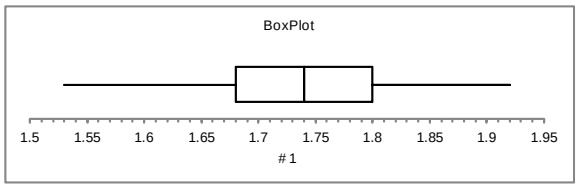


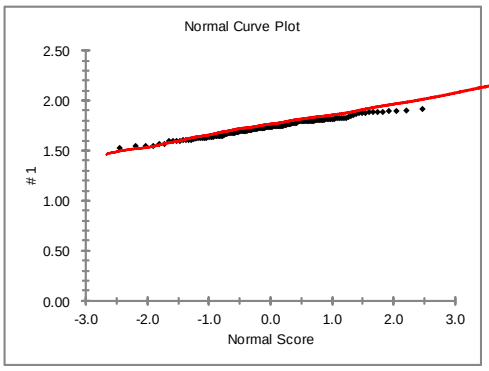
c) Normal distribution

Ex. Heights data

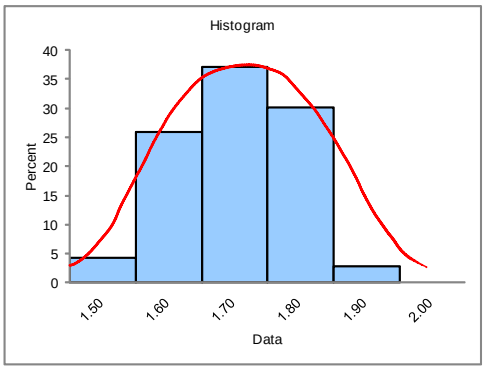
http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Q600-2013-Scanned-Height-Gender-Handspan.pdf

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Q600-2013-Height-Gender-Handspan.xlsx



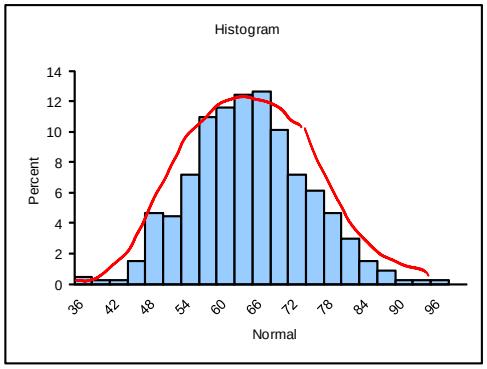


Line



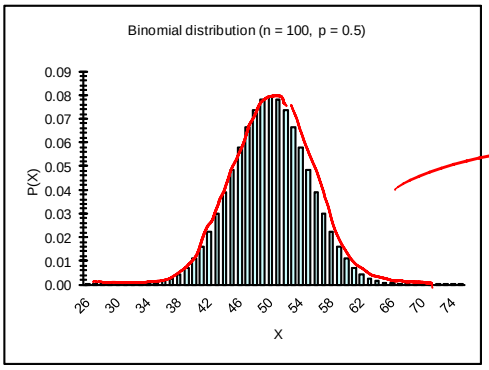
Ex Test scores

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/NormalData-475_000.xls



Ex. Binomial with large n (100) and $p \simeq .5$

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Binomial-Normal.xls



normal

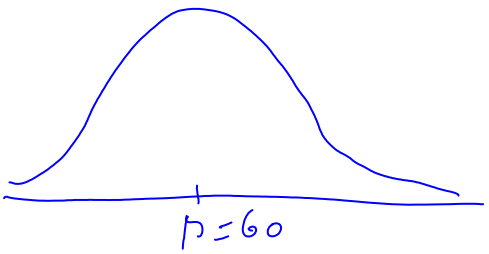
Ex. Galton's board

http://mathworld.wolfram.com/GaltonBoard.html

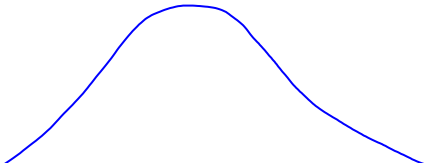
http://www.youtube.com/watch?v=xDIyAOBa_yU

Q: What's the effect of μ & σ on distribut.?

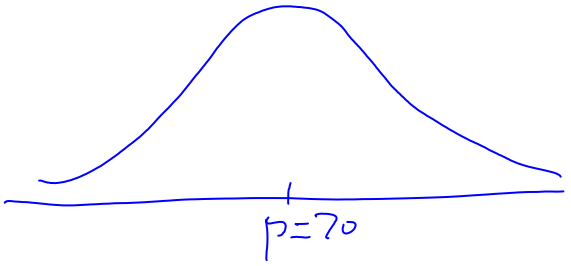
Ex



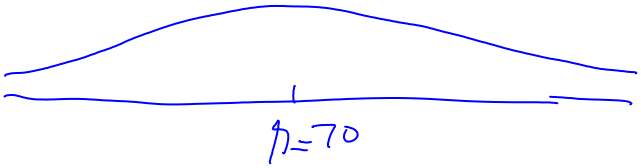
$\mu = 60$, $\sigma = 3$



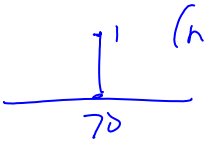
$\mu = 70$, $\sigma = 3$



$\mu = 70, \quad \sigma = 3$



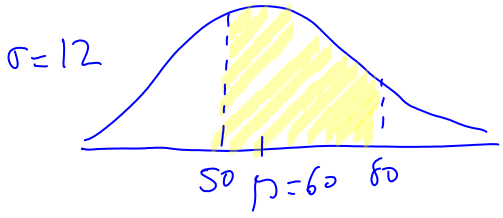
$\mu = 70, \quad \sigma = 6$



$\mu = 70, \quad \sigma = 0$ (no variation)

μ: characterizes location
σ: " dispersion

(i) Calculating Probabilities



$Pr(50 \leq X \leq 80) = ?$

No help from Empirical Rule

Recall z-score from Ch. 2

$$z = \frac{x-\mu}{\sigma}$$

X normal with μ & σ

$$Z = \frac{X-\mu}{\sigma}$$

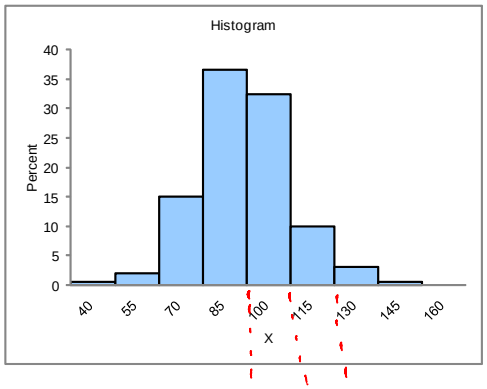
Fact

X normal with mean μ and sd. σ

Z " " " 0 " " 1

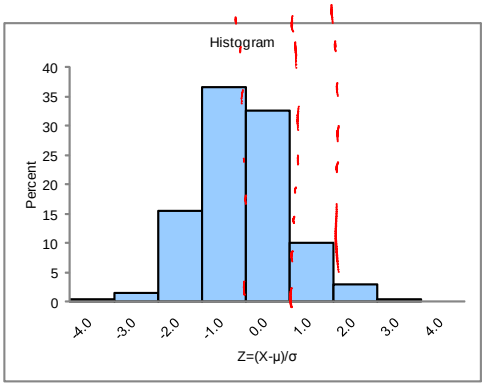
Demo

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/IQ-Scores-200-2009.xlsx



	X
count	200
mean	99.0955
sample variance	233.0254
sample standard deviation	15.2652
minimum	47.65
maximum	148.69
range	101.04

Pasted from <file:///C:\DOCUME~1\parlar\LOCALS~1\Temp\IQ-Scores-200-2009.xlsx>



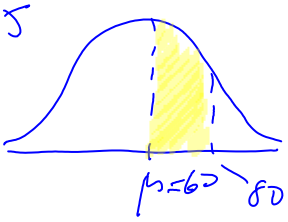
	Z=(X-μ)/σ
count	200
mean	-0.0603
sample variance	1.0357
sample standard deviation	1.0177
minimum	-3.49
maximum	3.246
range	6.736

Pasted from <file:///C:\DOCUME~1\parlar\LOCALS~1\Temp\IQ-Scores-200-2009.xlsx>

Z is called standardized normal

Ex. Test scores $\mu = 60$, $\sigma = 15$

$Pr(60 \le X \le 80) = ?$



Standardize

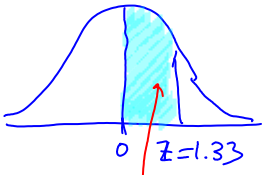
$60 < X < 80$

Standardize

$$60 \le X \le 80$$

$$\frac{60-60}{15} \le \frac{X-60}{15} \le \frac{80-60}{15}$$

$$0 \le Z \le 1.33$$



Find $Pr(0 \le Z \le 1.33) =$

Table 5.1 (p.156)
" A.3 (p.651)

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/NormalTable.wmf

Z	0.00	0.01	0.03	..	0.09
0.0					
0.1					
:					
1.1					
1.2					
1.3			.4082		
:					

$\Rightarrow$ $Pr(60 \le X \le 80) = .4082$ also

Ex. $Pr(50 \le X \le 90) = ?$

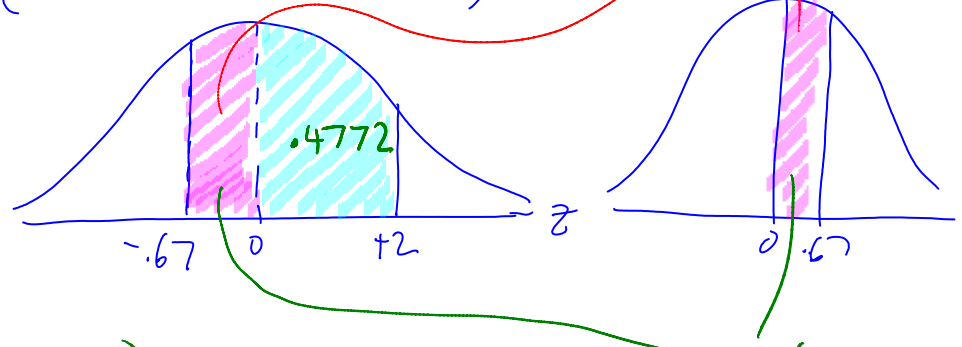
$$50 \le X \le 90$$

$$\frac{50-60}{15} \le \frac{X-60}{15} \le \frac{90-60}{15}$$

same

$Pr(-.67 \le Z \le 2) = ?$

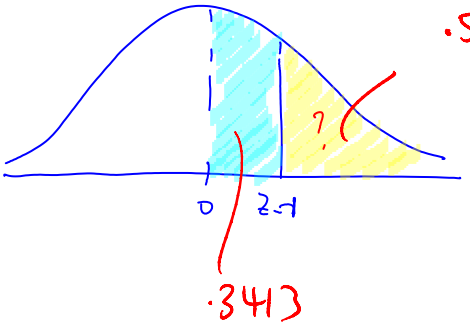
same



.2486

$Pr(-.67 \le Z \le 2) = .4772 + .2486$
$= .7258 = Pr(50 \le X \le 90)$

Ex. $Pr(Z > 1) = ?$

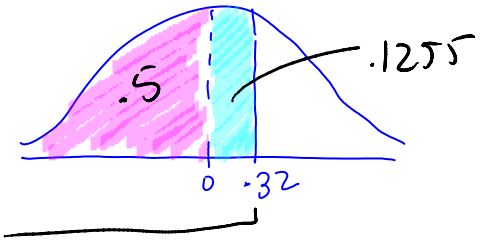


.5 − .3413
= .1587

.3413

Ex. $Pr(Z \le .32) = ?$

.5 + .1255 = .6255



Cumulative table

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Table-A4.pdf

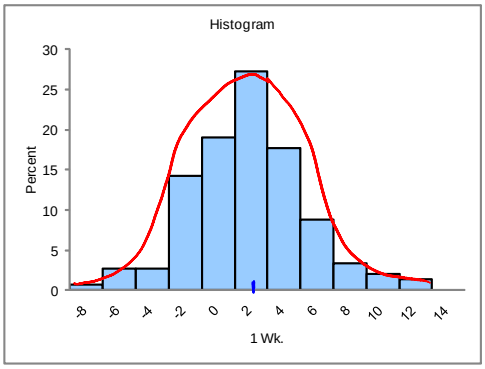
Ex. S&P 500 Index

11 sectors → 147 subsectors

http://www.standardandpoors.com/indices/sp-500/en/us/?indexId=spusa-500-usduf--p-us-l--

http://www.spindices.com/documents/additional-material/gics-500-scorecard.pdf

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/GICS_500_Scorecard-1.xlsx



$Pr(X \le 0)$

$\mu = 2.765\%$

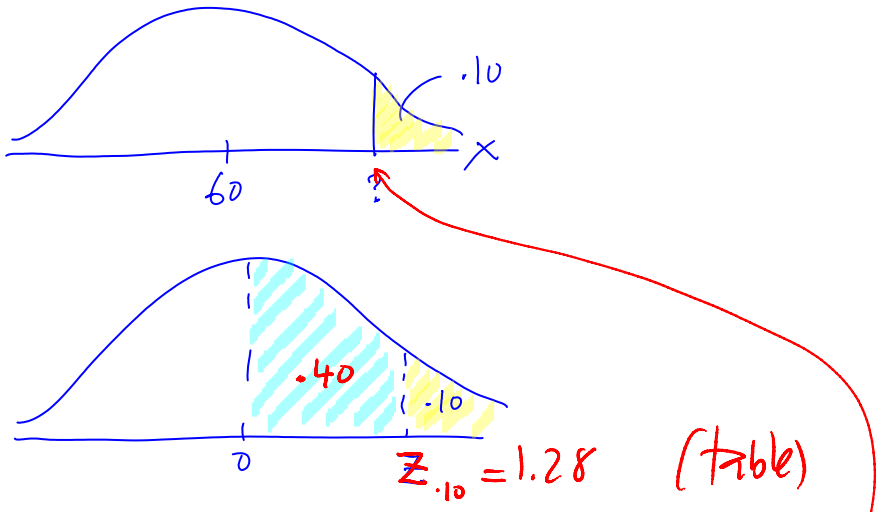
$\sigma = 3.444\%$

$$Pr(X \le 0) = Pr\left(\frac{X - 2.765}{3.444} \le \frac{0 - 2.765}{3.444}\right)$$

$$= Pr(Z \le -.803) = 0.2110$$

(ii) Calculating the z-score

Ex. Exam $\mu = 60$, $\sigma = 15$



$$X = \mu + \sigma \cdot Z_{.10} = 60 + (15)(1.28)$$

$$= 79.2$$