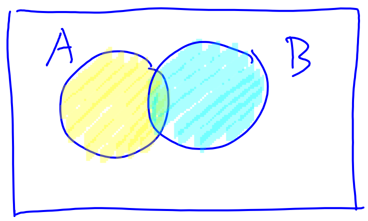
2013-10-03-C02

Thursday, October 03, 2013
10:44 AM

(2) Union & intersection

A

B



intersection

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

union

Ex. Two dice

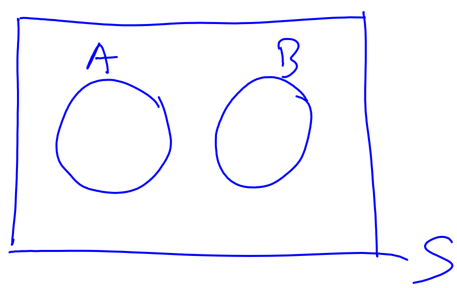
$A = \{11, 22, 33, 44, 55, 66\}$ Both sides same

$B = \{46, 55, 64\}$ Sum = 10

$A \cup B = \{11, 22, 33, 44, 55, 66, 46, 64\}$

$A \cap B = \{55\}$

(4) Mutually exclusive events



Ex. Two dice

A = same on both

B = sum equals 3

d) Joint, marginal & conditional prob's

Ex. Product preference

1000 surveyed (sampled)

		Preference		
		Coke (c)	Pepsi (p)	Total
Gender	Male (M)	200	300	500
	Female (F)	100	400	500
		300	700	1,000

Joint

$$Pr(M \cap C) = \frac{200}{1{,}000} = .2$$

$$Pr(F \cap P) = \frac{400}{1000} = .4$$

etc

Marginal

$$Pr(C) = \frac{300}{1000} = .3$$

etc

$$Pr(M) = \frac{500}{1000} = .5$$

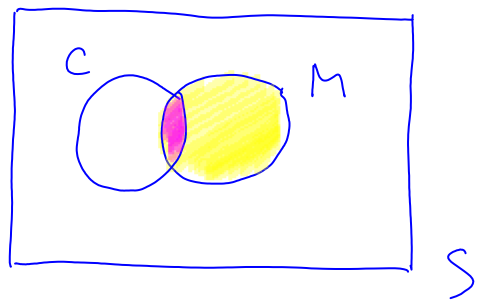
Summary

	C	P	Marginal
M	.2	.3	.5
F	.1	.4	.5
Marginal	.3	.7	1.0

Conditional

$$Pr(C \mid M) = Pr(\text{prefers Coke given male})$$

given



$$Pr(C|M) = \frac{Pr(C \cap M)}{Pr(M)}$$

$$Pr(C|M) = \frac{Pr(C \cap M)}{Pr(M)} = \frac{.2}{.5} = .4$$

$$Pr(C|F) = \quad = \frac{.1}{.5} = .2$$

etc

Summary

		C	P	Total
given	M	.4	.6	1.
	F	.2	.8	1.

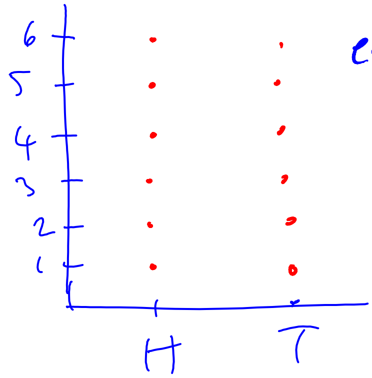
e) Independence of events

A and B are independent if

$$Pr(A|B) = Pr(A)$$

Ex. Die & Coin

die



each $\frac{1}{12}$

$Pr(H \mid die = 4) = \frac{1}{2}$

$Pr(H) = \frac{1}{2}$

Note $\quad Pr(A \mid B) = \dfrac{Pr(A \cap B)}{Pr(B)}$

$\Rightarrow \quad Pr(A \cap B) = \underbrace{Pr(A \mid B)}_{Pr(A)} Pr(B)$

$\Rightarrow \quad Pr(A \cap B) = Pr(A)\, Pr(B)$

Ex Coin & die

$Pr(H \cap 4) = Pr(H) \cdot Pr(4)$

$= \frac{1}{2} \cdot \frac{1}{6} = \frac{1}{12}$

h2

Ex. Product preference

$Pr(C \mid M) = .4$, $\quad Pr(C) = .3$

C & M are not independent

Ex. Lady Gaga on iPod

100 songs $\rightarrow$ shuffle

L: Lady Gaga on 1st song

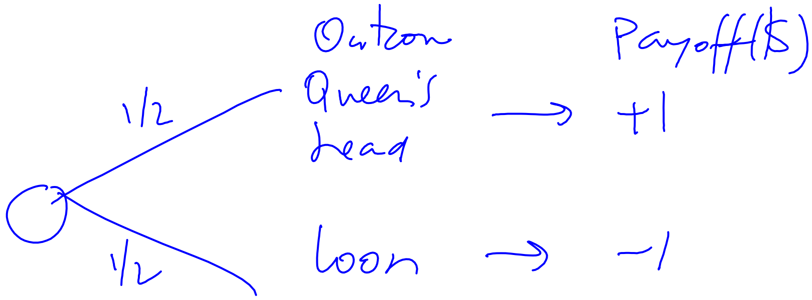
$Pr(L) = \frac{1}{100}$, $\quad Pr(\underbrace{\bar{L}}_{\text{not } L}) = \frac{99}{100}$

3 shuffles

$$Pr(\text{at least one } L) = 1 - Pr(\text{no } L)$$
$$= 1 - \left(\frac{99}{100} \cdot \frac{95}{100} \cdot \frac{99}{100}\right)$$
$$= 1 - \left(\frac{99}{100}\right)^3 \simeq 0.03$$

Ch. 4 Discrete random variables

a) What is a random variable?



Def A random variable associates a numerical value with each outcome

X : \$ payoff as a result of tossing coin

Ex. Mold car tires in pairs

#1	#2	Total defect
G	G	0
G	D	1
D	G	1
D	D	2

X: # defective tires in pair : 0, 1, 2

b) Discrete prob. distributions

Ex. Two tires

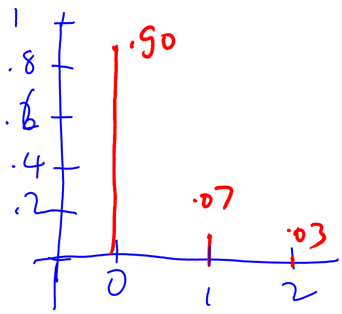
$Pr(0 \text{ defects}) = .90 = p(0)$

$Pr(1 \quad '' \quad) = .07 = p(1)$

$Pr(2 \quad '' \quad) = .03 = p(2)$

x	p(x)
0	.90
1	.07
2	.03
	1.00

prob. dist for X



Prod. continues for 100 runs (200 tires) $\xrightarrow{}$ 2 tires/run)

		Total
Approximately	90 out of 100 have 0 defects	0
	7 " " 100 " 1	7
	3 " " 100 " 2	6
		13

$$\text{Avg. \# defectives/run} = \frac{13}{100} = .13$$

$$\text{'' '' ''/tire} = \frac{13}{200} = .065$$

Ex. Bicycle sales (Pierik's)

Last 100 business days

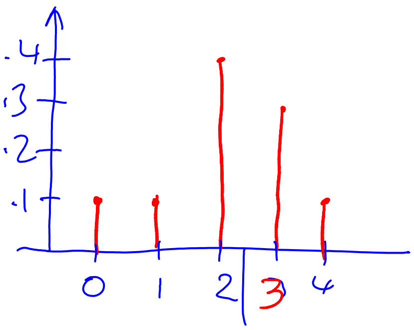
#sold (x)	#days	p(x)	Total sales

0	10	.1	0
1	10	.1	10
2	40	.4	80
3	30	.3	90
4	10	.1	40
	100		220

Avg. # sals/day $= \frac{220}{100} = 2.2$

Mean (expected) value of X

$$E(X) = \mu_X = \sum_{\text{all } x} x\, p(x)$$



$\mu_X = 2.2$

h3

x	$p(x)$	$x\,p(x)$
0	.1	0
1	.1	.1
2	.4	.8
3	.3	.9
4	.1	.4
		$\sum = 2.2$

Ex. Home insurance policy

$\underbrace{\$500,000}_{\text{worth}}$ $\underbrace{\$400/\text{yr}}_{\text{premium}}$

Outcome	Profit (x)	$P(x)$
Fire	$400 - 500,000 = -499,600$	.0001
No fire	400	.9999

$$\mu_X = .0001(-499,600) + .9999(400)$$
$$= \$350 \quad \text{(single home)}$$

$10,000 \times 400 = 4,000,000$

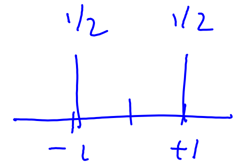
$$\frac{-500{,}000}{3{,}500{,}000} \div 10{,}000 = \$350/\text{home}$$

Variance of a random variable X

Pop'n. $\sigma^2 = \frac{1}{N}\sum_{i=1}^{N}(x_i - \mu)^2$

Rand. var. $\sigma_X^2 = \sum_{\text{all } x}(x - \mu_x)^2 \cdot p(x)$

Ex. Coin flip



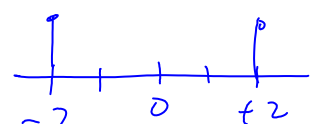
	x	$p(x)$	$(x-\mu_x)^2$	$(x-\mu_x)^2 p(x)$
H	+1	1/2	1	1/2
T	-1	1/2	1	1/2

$\Sigma = 1 = \sigma_X^2$

$\mu_X = \frac{1}{2}(+1) + \frac{1}{2}(-1) = 0$

$\sigma_X^2 = 1$, $\sigma_X = \sqrt{\sigma_X^2} = 1$

Ex. +2, -2 case



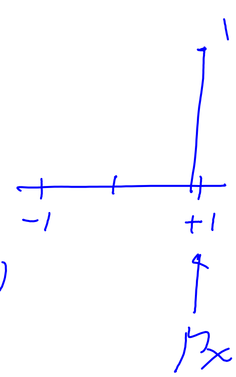
	x	$p(x)$	$(x-\mu_x)^2$	$(x-\mu_x)^2 p(x)$
H	+2	1/2	4	2
T	-2	1/2	4	2

$\Sigma = 4 = \sigma_X^2$

$\mu_X = 0$

$\sigma_X^2 = 4$, $\sigma_X = 2$

Ex Loaded coin (H always)



	x	$p(x)$	$(x-\mu_x)^2$	$(x-\mu_x)^2 p(x)$
H	+1	1	0	0
T	-1	0	4	0

$\Sigma = 0 = \sigma^2$

$\sum = 0 = \sigma_X^2$

$\mu_X = +1(1) + (-1)\cdot 0 = 1$

$\sigma_X^2 = 0$

c) Binomial distribution

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Binomial.pdf