2013-10-01-EC01

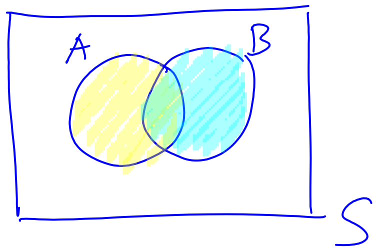
Tuesday, October 01, 2013
6:41 PM

(3)

(2) Union & Intersection

A

B



intersection

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

union

Ex. Two dice

$A = \{11, 22, 33, 44, 55, \ldots, 66\}$ same both

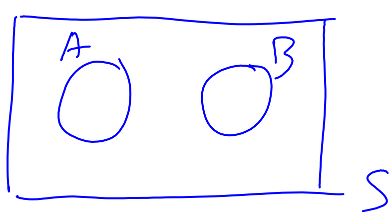
$B = \{46, 55, 64\}$ sum=10

$A \cap B = \{55\}$

(4) $Pr(S) = 1$. $Pr(H \text{ or } T) = 1$

$Pr(1 \text{ or } 2 \text{ or } 3 \ldots \text{ or } 6) = 1$

(5) Mutually exclusive events



Ex. $A = \{HH, TT\}$, $Pr(A) = \frac{1}{2}$

$B = \{HT\}$, $Pr(B) = \frac{1}{4}$

$Pr(A \cap B) = 0$

d) Conditional, joint & marginal prob's

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Ex. Product preference (marketing)

1000 Surveyed

		Preference		
		Coke (C)	Pepsi (P)	Total
Gender	Male (M)	200	300	500
	Female (F)	100	400	500
		300	700	1,000

Joint

$$Pr(M \cap C) = \frac{200}{1000} = .2$$

$$Pr(F \cap C) = \frac{100}{1000} = .1 \qquad \text{etc}$$

Marginal

$$Pr(M) = \frac{500}{1000} = .5 \qquad \text{etc}$$

$$Pr(C) = \frac{300}{1000} = .3$$

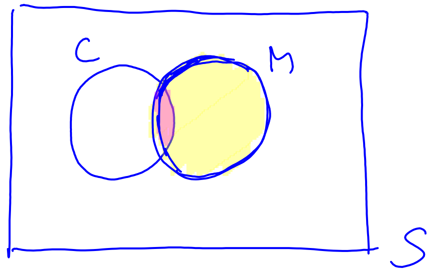
Summary

	C	P	Marginal
M	.2	.3	.5
F	.1	.4	.5
Marginal →	.3	.7	

Conditional $\quad Pr(C \mid M) = Pr($prefers Coke given male)

given

Venn



Pr

$$\left.\begin{aligned}
Pr(C|M) &= \frac{Pr(C\cap M)}{Pr(M)} = \frac{.2}{.5} = .4\\
Pr(P|M) &= \qquad\qquad\qquad\quad = .6\\
Pr(C|F) &= \qquad\qquad\qquad\quad = .2\\
Pr(P|F) &= \qquad\qquad\qquad\quad = .8
\end{aligned}\right\}$$

e) Independence of events

A and B are independent if

$$Pr(A|B) = P(A)$$

Ex. A: roll 4 (die)

B: Throw H (coin)

$$Pr(A|B) = Pr(A) = \frac{1}{6}$$

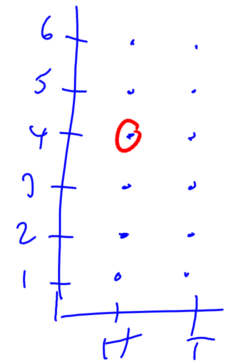
Note: $$Pr(A|B) = \frac{Pr(A\cap B)}{Pr(B)}$$

$$\Rightarrow \quad Pr(A\cap B) = \underbrace{Pr(A|B)}_{Pr(A)} \cdot Pr(B)$$

$$\Rightarrow \quad Pr(A \cap B) = Pr(A) \cdot Pr(B)$$

Ex. $Pr(4 \cap H) = Pr(4) \cdot Pr(H)$

$= \frac{1}{6} \cdot \frac{1}{2} = \frac{1}{12}$



Ex. Product Preference

$Pr(C \mid M) = .4, \quad Pr(C) = .3$

Ex. Lady Gaga on iPod

100 songs

L: Lady Gaga song on first song

$Pr(L) = \frac{1}{100}, \quad Pr(\bar{L}) = Pr(\text{not } L) = \frac{99}{100}$

Three shuffles

$Pr(\text{at least one } L) = 1 - Pr(\text{no } L)$

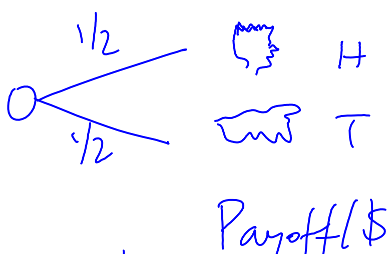
$= 1 - \frac{99}{100} \cdot \frac{99}{100} \cdot \frac{99}{100} = .03$

Ch.4. Discrete random variables

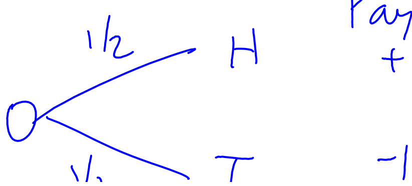
a) What is a random variable?

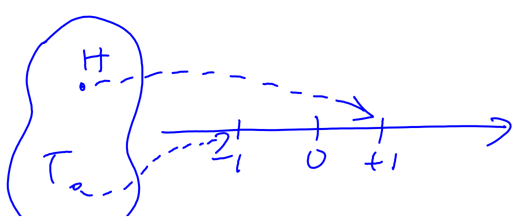
Ex. Coin

Outcome



Payoff($)





1/2 T -1

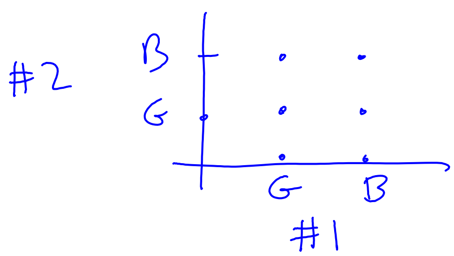
Def A random variable associates a <u>numerical</u> value with each outcome of an experiment

X: \$ payoff (+1 or -1) as a result of coin toss
discrete

Ex: X: # customers entering bookstore in one day (discrete 0, 1, 2, …)

Ex. X: # minutes you wait for bus (contin. ≥ 0)

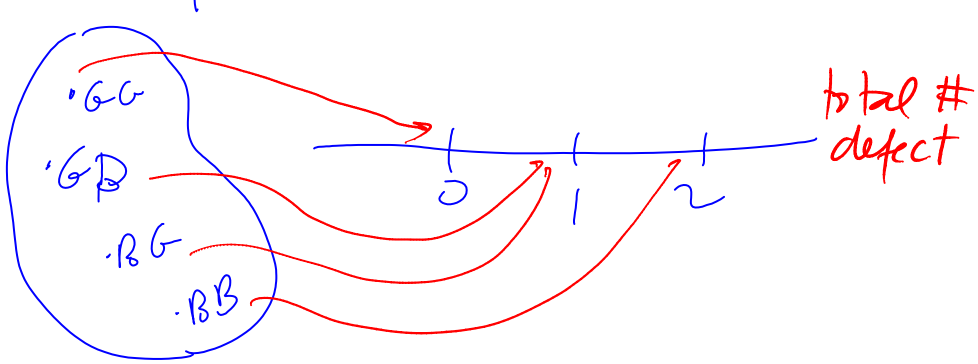
Ex. Mold tires in pairs



#1	#2	Total Bad
G	G	0
G	B	1
B	G	1
B	B	2

X: # defectives in a pair $= \{0, 1, 2\}$

Each run → one pair



b) Discrete Prob. distribution

Ex. Two tires

$Pr(0 \text{ def}) = 0.90 = p(0)$

$Pr(1 \ \ ") = 0.07 = p(1)$

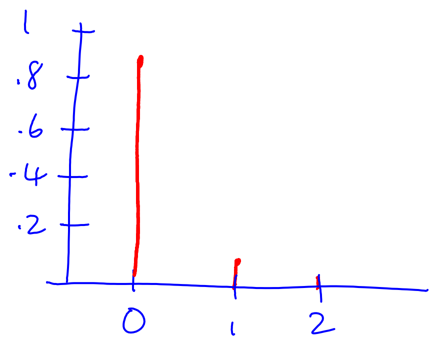
$Pr(2 \ \ ") = \underline{0.03} = p(2)$

$\qquad\qquad 1.00$

x	p(x)
0	0.90
1	0.07
2	0.03

1.00

prob. distribution



Suppose prod. continues for 100 runs (200 tires)

		Total def
Approx	90 out of 100 have 0 defect	0
	7 " " 100 " 1 "	7
	3 " " 100 " 2 "	6
		13

$.13 = \frac{13}{100}$: avg. # defectives / run

Ex. Bicycle sales (Pierikis)

Last 100 days

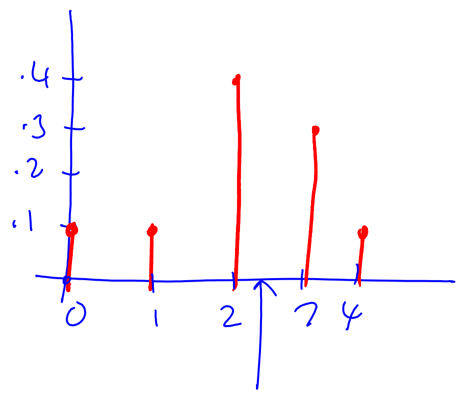
# sold x	# days	p(x)	Total sales

0	10	.1	0
1	10	.1	10
2	40	.4	80
3	30	.3	90
4	10	.1	40
	100		220

Avg. sales/day $= \frac{220}{100} = 2.2$

Mean (expected) value of X

$$E(X) = \mu_X = \sum_{\text{all } x} x\,p(x)$$



x	$p(x)$	$x\,p(x)$
0	.1	0
1	.1	.1
2	.4	.8
3	.3	.9
4	.1	.4
		$\sum = 2.2$

$E(X) = \mu_X = 2.2$

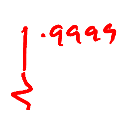
Ex. Home Insurance Policy

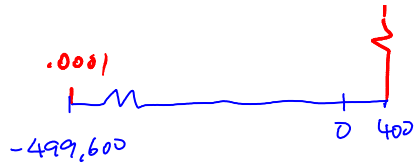
Insure your home for \$500,000 & pay a premium of \$400/yr

Q: What's insurer's expected profit?

Outcome	Profit (x)	Prob
Fire	$400 - 500{,}000 = -499{,}600$	.0001
No fire	400	.9999

X: company's profit





$E(X) = \mu_X = .0001 \times (-499{,}600) + .9999 \times (400)$

$= \$350$ ← risky if one home

# homes	× prem.	
10,000	× 400	= 4,000,000

$\ -500{,}000$

$3{,}500{,}000 \div 10{,}000 = 350$

Variance, etc. for rand. var. X

Recall $\sigma^2 = \frac{1}{N} \sum_{i=1}^{N} (x_i - \mu)^2$ (Pop'n)

For X $\sigma_X^2 = \sum_{\text{all } x} (x - \mu_X)^2 p(x)$

$\sigma_X = \sqrt{\sigma_X^2}$

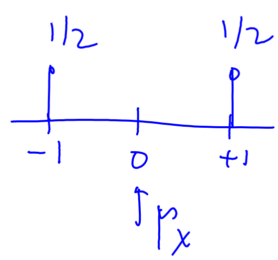
Ex. Coin flip

Outcome	Payoff x	p(x)	$(x-\mu_X)^2$	$(x-\mu_X)^2 p(x)$
H	+1	1/2	1	1/2
T	-1	1/2	1	1/2
				1

$\mu_X = \frac{1}{2}(+1) + \frac{1}{2}(-1) = 0$ fair game

$\sigma_X^2 = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 1 = 1$

$\sigma_X = 1$

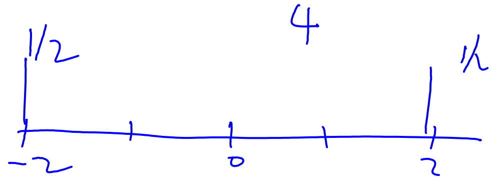


Ex. Higher risk/reward

	x	$p(x)$	$(x-\mu_x)^2$	$(x-\mu_x)^2 p(x)$
H	+2	1/2	4	2
T	−2	1/2	4	2
				4

$\mu_x = \frac{1}{2}(2) + \frac{1}{2}(-2) = 0$

$\sigma_x^2 = 4 \rightarrow \sigma_x = 2$

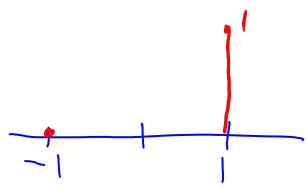


Ex. Loaded coin toss

	x	$p(x)$	$(x-\mu_x)^2$	$(x-\mu_x)^2 p(x)$
H	+1	1	0	0
T	−1	0	4	0
				0

$\mu_x = 1\cdot(1) + \cancel{(-1)}\cdot 0 = 1$

$\sigma_x^2 = 0$



c) Binomial distribution

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