2013-09-24-EC01

Tuesday, September 24, 2013
6:24 PM

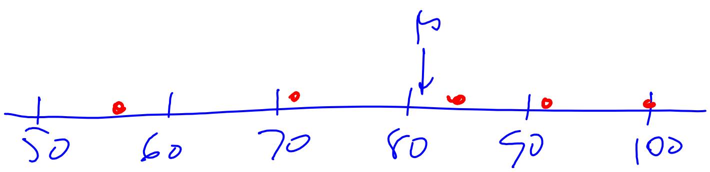
$$x = \mu + z\sigma$$

$$\frac{x-\mu}{\sigma} = z$$

Ex. Test scores in a Ph.D. class

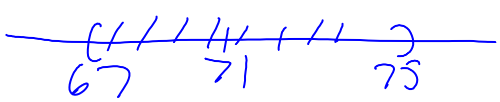
x: 56, 72, 83, 92, 100

$\mu = 80.60$
$\sigma = 15.44$



x	$x-\mu$	$z=(x-\mu)/\sigma$
56	-24.6	-1.6
72	-8.6	-0.6
83	2.4	0.2
92	11.4	0.7
100	19.4	1.3

Ex. Quality improvement



Acceptable range
(71 ∓ 4)

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/CoffeeTemp-Tolerance.xls

73
76
69
67
74
70
69
72
71
68
75
72
67
74
72
68
70
77
68
72
69
75
68

"capable"?

73

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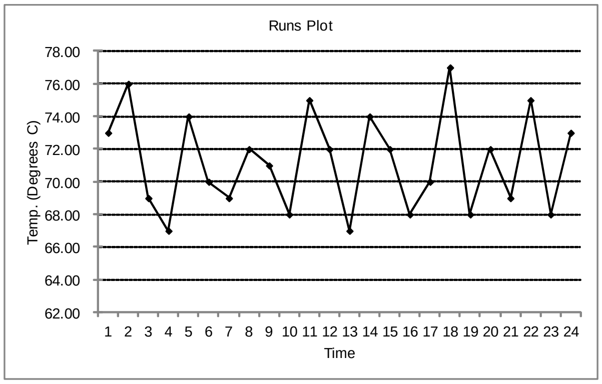

count	24
mean	71.21
sample variance	8.87
sample standard deviation	2.98
minimum	67
maximum	77

$n = 24$
$\bar{x} = 71.21$
$s = 2.98$

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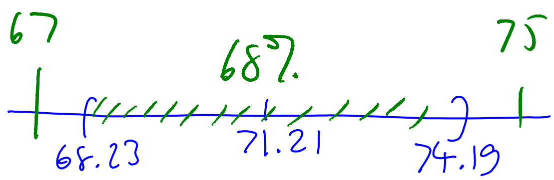
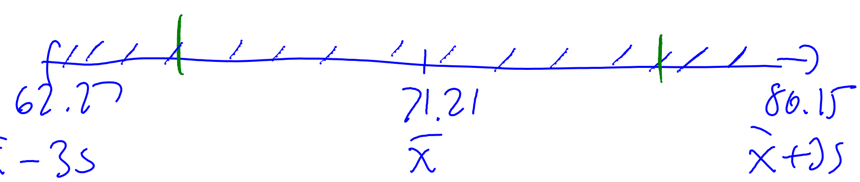
~68%
$(\bar{x} \mp 1.s)$



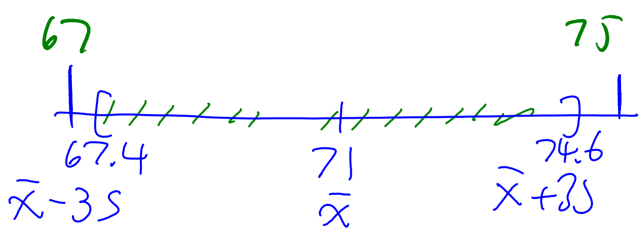
~99%



Reduce variability (s)

New sample $\bar{x} = 71$, $s = 1.2$

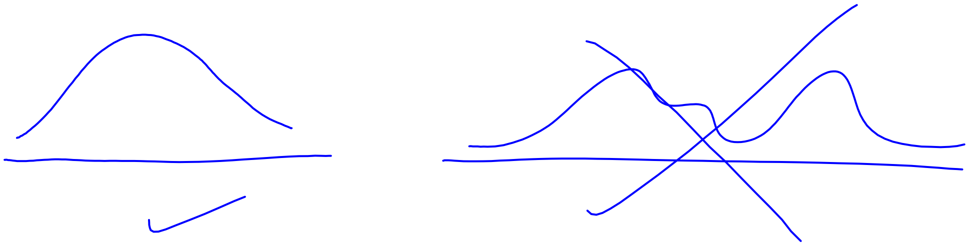
99%



"capable"

Remark: Above technique applies only for

Single mode distributions



d) More measures of variation

Ex. GMAT score 660 → 83rd percentile

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/GMAT-Percentiles.pdf

http://www.testmasters.net/GmatAbout/Scoring-Scale

http://www.platinumgmat.com/about_gmat/gmat_score_breakdown

The **pth percentile** of a group of *n* measurements is a value such that (approximately) *p*% of measurements fall at or below the value and (approximately) (100-*p*)% fall at or above the value

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Median
Md: 50th percentile

Ex. Exam scores

$n=8$

			56		74		83	
Scores	36	40	72	74	74	80	86	90
Posit.	1	2	3	4	5	6	7	8

Pos. of $p=25$: 2.5
" $p=50$: 4.5
" $p=75$: 6.5

One (36) drops out

$n=7$

	14%	28%		74			
Scors	40	72	74	74	80	86	90
Pos	1	2	3	4	5	6	7

Pos. $p=25$: 2
$p=50$: 4
$p=75$: 6

An easy method for locating the position of the *p*th percentile of *n* measurements:

- First, order all measurements and calculate $i=(p/100)*n$.
- If *i* is an integer, then the position is the average of measurements in positions *i* and *i+1*.
- If *i* is not an integer, then the position is the next integer greater than *i*.

Pasted from <http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/ch-02.html>

$p=25$, $n=7$

$i=\frac{p}{100}\cdot n=\frac{25}{100}\cdot 7=1.75$

posit $=2 \rightarrow 72$

- If i is not an integer, then the position is the next integer greater than i.

Pasted from <http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/ch-02.html>

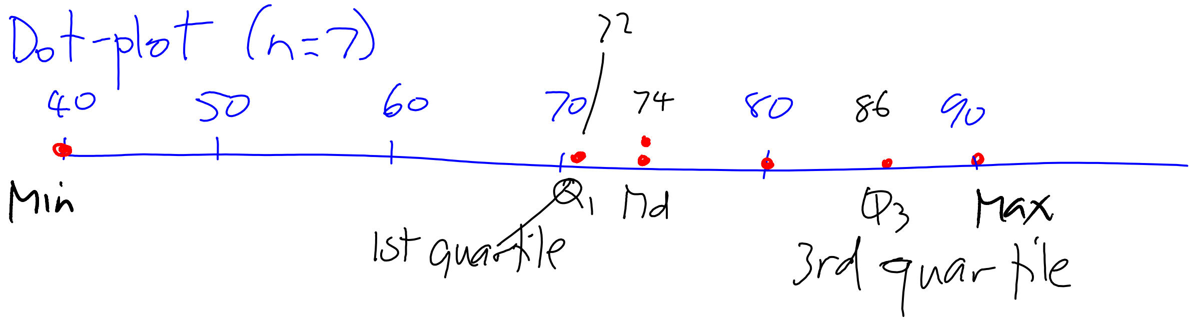
pos it = 2 → 72

$p = 75$, $i = .75 \times 7 = 5.25 \to 6 \to 86$

$n = 8$, $p = .25$, $i = .25 \times 8 = 2$

$pos = \frac{1}{2}(2+3) = 2.5 \to 56$

Dot-plot (n=7)

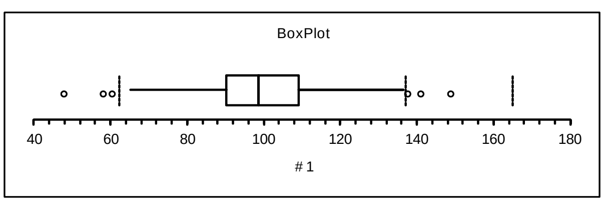


Interquartile range $= Q_3 - Q_1 = 86 - 72 = 14$

Box and whisker plot

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Box-Whisker.pdf

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/IQScores-500-Boxplot.xls



Ch. 3 Probability

a) Basic concepts

Ex. Coin toss

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/CoinToss_000.xls

Ex. 6/49

Ex. "Baby" lottery 2/4

1	2	3	4	
1	2			–
1		3		–
1			4	–
	2	3		–
	2		4	–
		3	4	–

$Pr(win) = \frac{1}{6}$

6/49 $\quad Pr(win) = \frac{1}{13{,}983{,}816} = .00000072$

Ex.
Birthday

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/1.DB-2013-EC01.pdf

http://en.wikipedia.org/wiki/Birthday_paradox

Ex. Monty Hall's car + goats pbln

http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/Monty-Solution.pdf

Pick #1 & always switch

Methods

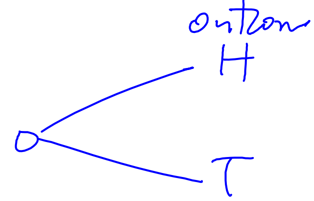
Classical
- Outcomes are equally likely

Relative freq
(empirical)

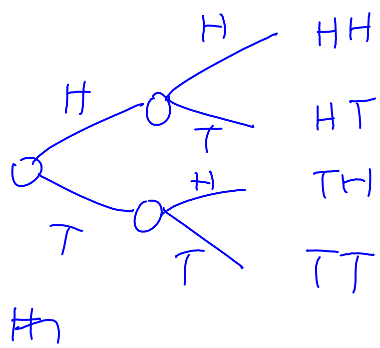
Subjective

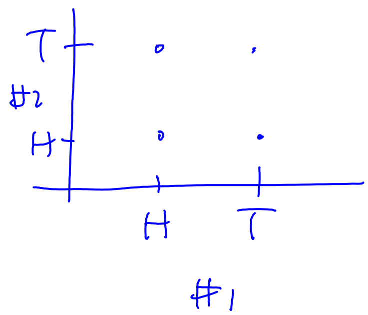
Random experiment : uncertain outcomes

Ex. Coin toss



Ex. Two coins



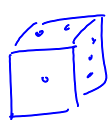


b) Sample spaces & events

Def. Set of all possible outcomes of a random exp is sample space (S)

Ex. Coin $S = \{H, T\}$, $Pr(H) = Pr(T) = \frac{1}{2}$

Ex. One die



$S = \{1, 2, \ldots, 6\}$

Ex. Two coins $S = \{HH, HT, TH, TT\}$

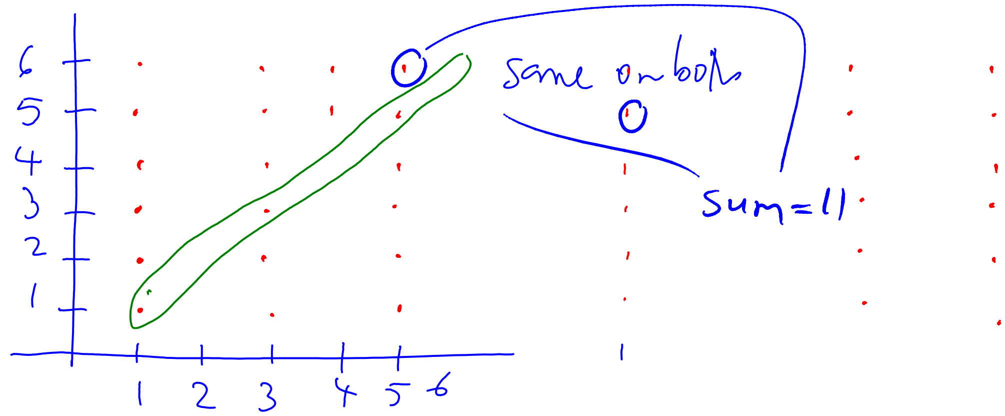
$Pr(HH) = \frac{1}{4}$

Ex Backgammon

http://en.wikipedia.org/wiki/Backgammon

$S = \{(1,1), (1,2), \ldots, (6,6)\}$

#2



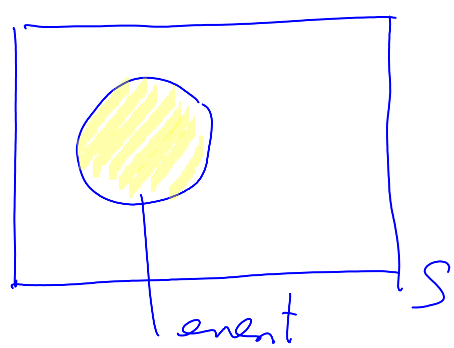
$Pr(\overbrace{\text{same on both}}^{A}) = \frac{6}{36} = \frac{1}{6} = 0.167$

$Pr(\underbrace{\text{sum} = 11}_{B}) = \frac{2}{36} = 0.055$

$Pr(A \text{ or } B) = \frac{8}{36} = .167 + .055 = .222$

<u>Def</u>. An <u>event</u> is a collection of outcomes

Venn diagram



$A = \{\text{same on both}\}$
$B = \{\text{sum} = 11\}$

When outcomes are equally likely,

$$Pr(\text{event}) = \frac{\text{outcomes that make up event}}{\text{all outcomes}}$$

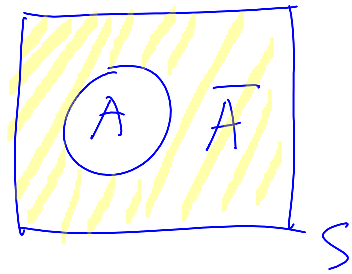
Ex. Lotto 6/49

Ex. Lotto 6/49

$$Pr(\text{win}) = \frac{\text{your 6 \#s}}{\text{all poss outcoms}} = \frac{1}{13{,}983{,}816}$$

c) Rules to calculate prob's

(i) Complement



A = Some event

$\bar{A}$: Complement of A

$$Pr(\bar{A}) = 1 - Pr(A)$$

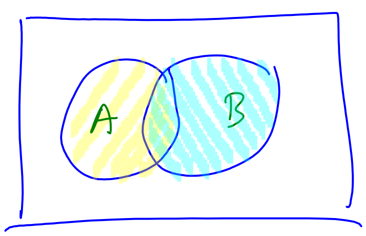
Ex. $A = \{HH, TT\}$, $\bar{A} = \{HT, TH\}$

$Pr(A) = \frac{1}{2}$, $Pr(\bar{A}) = 1 - \frac{1}{2} = \frac{1}{2}$

(ii) Union $\quad A \cup B$

A

B



Ex. Two dice

$A = \{11, 22, 33, 44, 55, 66\}$ Both same

$B = \{46, 55, 64\}$ Sum = 10

$A \cup B = \{11, 22, 33, 44, 55, 66, 46, 64\}$

$$Pr(A \cup B) = Pr(A) + Pr(B) - Pr(A \cap B)$$

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(A \cap B)$$

$$\Pr(A) = \frac{6}{36}, \quad \Pr(B) = \frac{3}{36}, \quad \Pr(A \text{ and } B) = \frac{1}{36}$$