

Ch.2 Descriptive Statistics

C. Minard

http://en.wikipedia.org/wiki/Charles_Joseph_Minard

a) Shape of a distribution

. Stem & leaf



173 \Rightarrow

Stem x10	Leaf x1
17	3

Ex. Test scores

70, 72, 76, 80, 84; 84, 88, 90, 90, 94; 96, 98, 100, 100, 100

Count	Stem (x10)	Leaf (x1)	
3	7	0 2 6	
4	8	0 4 4 8	
5	9	0 0 4 6 8	} A+
3	10	0 0 0	
15			

. Dot plots & histograms

<http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/TestScores/MBA-DotPlot.xls>

<http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/TestScores/MBA-DifferentWidths.xls>

width (length) : 1, 4, 10, 20

Which is best?

n: # data points

K: # classes

L: class length

$n \approx \dots \parallel \perp \llcorner \lrcorner \dashv \vdash \dashv \vdash \dots \dashv \vdash \dots \dashv \vdash \dots$

① Find smallest K such that $2^K > n$. Then,

② $\rightarrow L = \frac{\max - \min}{K}$

$n=15$

2^K vs. $n=15$

$K=1$

$2^1 = 2 \not> 15$

$K=3$

$K=4$

$2^4 = 16 > 15$

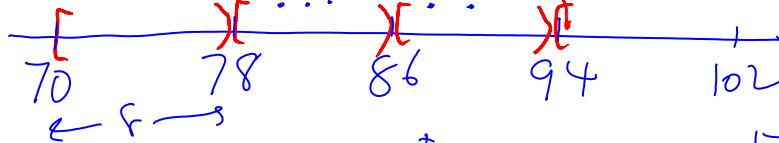
$L = \frac{100 - 70}{4} = 7.5 \rightarrow 8$
round up to

$K=4, L=8$

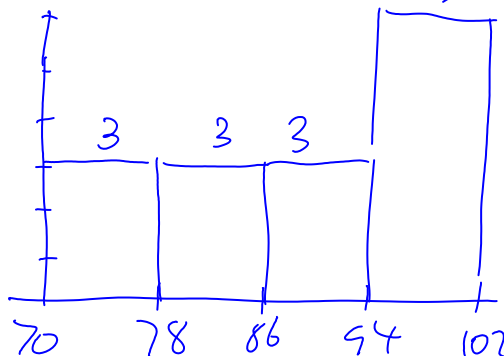
70, 72, 76, 80, 84; 84, 88, 90, 90, 94, 96, 98, 100, 100, 100.

Pasted from <<http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/ch-02.html>>

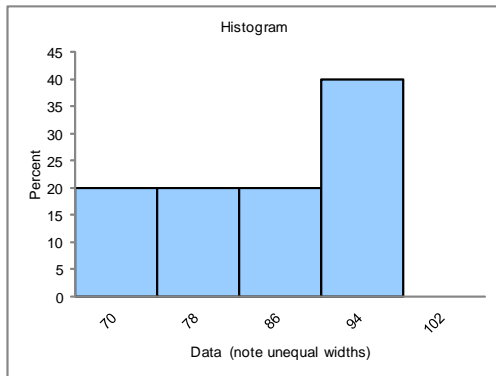
Next?



Class	Freq
70 to <78	3
78 " <86	3
86 " <94	3
94 " <102	6

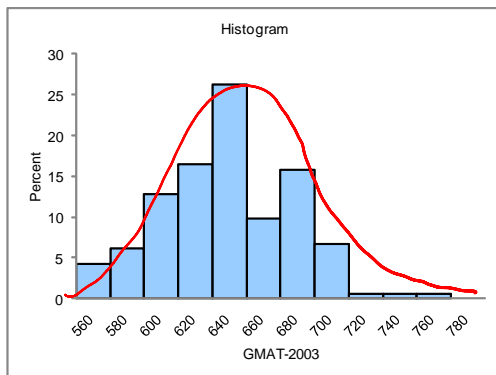


<http://profs.degroote.mcmaster.ca/ads/parlar/courses/q600/ChapterComments/documents/TestScoresMBA-Nice.xls>

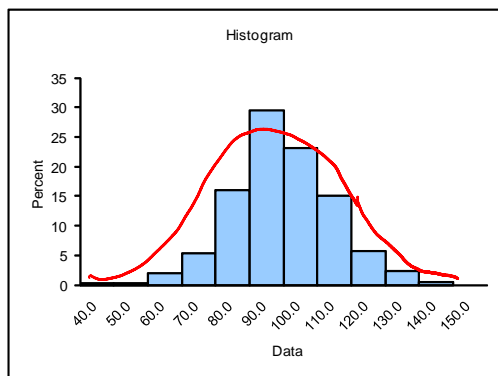


Ex. GMAT Scores 2003-2004-2005

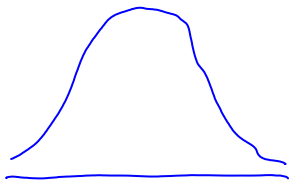
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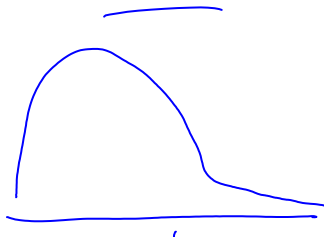
Ex 1Q



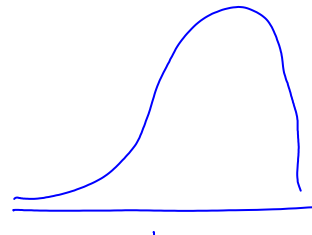
Ex Pb. 2.8, p. 37



Symmetric



positively-skewed



negatively-skewed

Ex Mac salaries

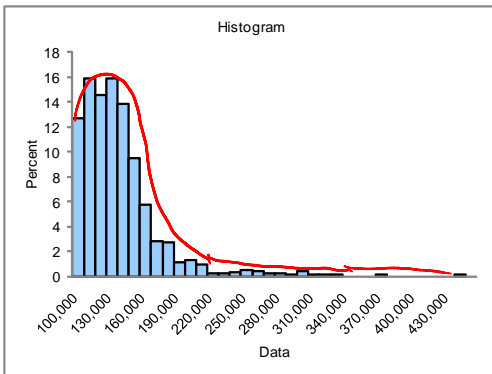
<http://profs.degroot.mcmaster.ca/ads/parlar/courses/g600/ChapterComments/documents/McMasterSalary-2011-for-2010.xlsx>

count	928
mean	140,885.0597
sample standard deviation	35,939.6578
sample variance	1,291,659,004.8693
minimum	100032.24
maximum	448977.4
range	348945.16

Pasted from <file:///C:/DOCUME~1/parlar/LOCALS~1/Temp/McMasterSalary-2011-for-2010.xlsx>

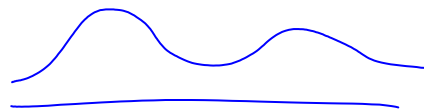
1st quartile	117,646.0025
median	134,052.8300
3rd quartile	151,663.5975
interquartile range	34,017.5950
mode	124,382.6300

Pasted from <file:///C:/DOCUME~1/parlar/LOCALS~1/Temp/McMasterSalary-2011-for-2010.xlsx>

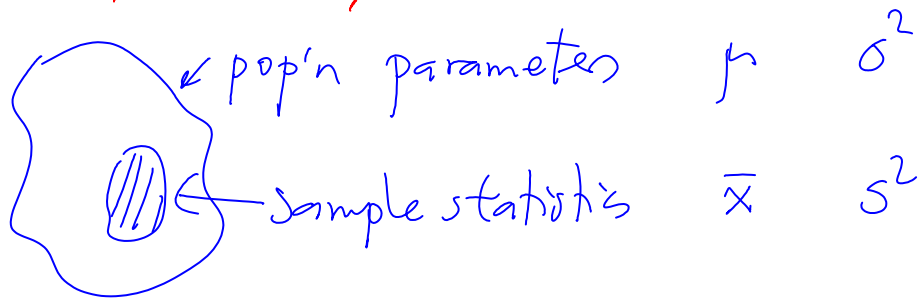


Positively skewed!

Heights



b) Mean, median, mode



• Mean (μ or \bar{x})

$N=15,$

70, 72, 76, 80, 84; 84; 88, 90, 90, 94; 96, 98, 100, 100, 100

Pasted from <<http://profs.degroote.mcmaster.ca/ads/paarl/courses/a600/ChapterComments/ch-02.htm>>

$$\mu = \frac{70+72+\dots+100}{15} = 88.13$$

$$\mu = \frac{X_1 + \dots + X_N}{N} = \frac{1}{N} \sum_{i=1}^N X_i$$

Sample of $n=3$

$$\bar{x} = \frac{70+72+80}{3} = 74$$

$$\bar{x} = \frac{X_1 + \dots + X_n}{n} = \frac{1}{n} \sum_{i=1}^n X_i$$

↑
sample statistic for population mean μ

• Median M_d

Ex. 6 people

Typist	A	36	(\$1,000)
Sec	B	42	
	C	30	

$n = 4 \times 167$

Sec	B	42	$\mu = \$82,167$
:	C	30	
:	D	40	
:	E	45	
President	F	300	

Rearrange

30
36
40 ← 41 = M_d
42
45
300

Median M_d : value above & below which lie an equal # of measurements

Ex. Mac $\mu = 141$
 $M_d = 134$

Mode M_o : most frequently occurring value in data

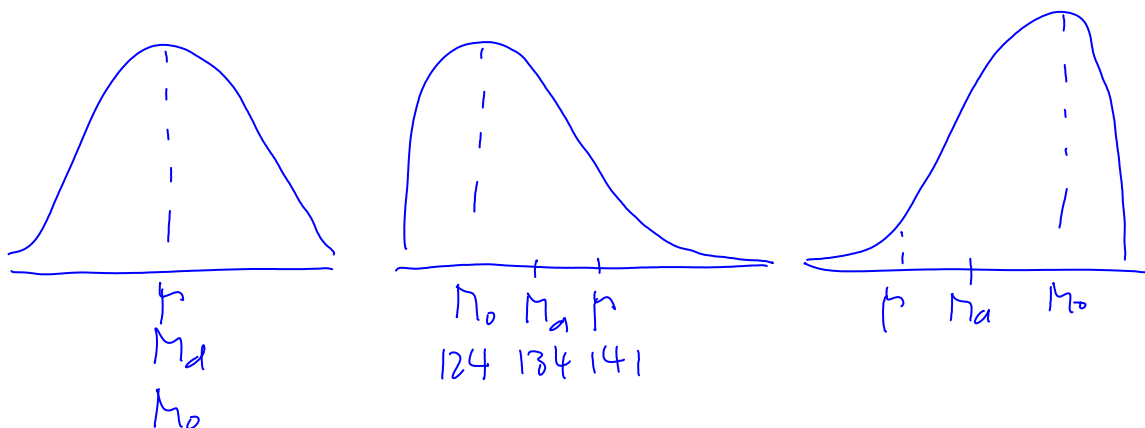
Ex. Test scores

70, 72, 76, 80, 84; 84; 88, 90, 90, 94; 96, 98, 100, 100, 100

$M_o = 100$

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Ex. Mac $M_o = 124$



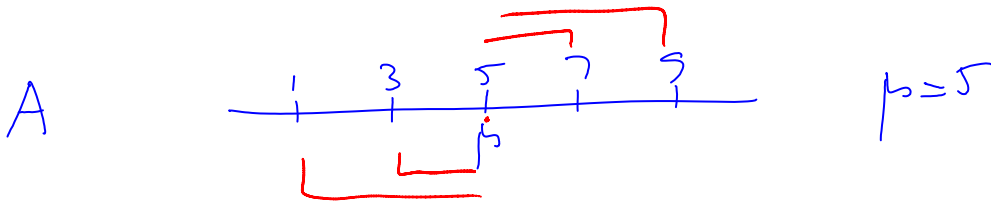
c) Range + Variance

Ex Three groups

		μ	M_d	M_o	Range
A	1, 3, 5, 7, 9	5	5	N/A	8
B	3, 5, 5, 5, 7	5	5	5	4
C	5, 5, 5, 5, 5	5	5	5	0
D	5 5 5 5 100				95
E	5 60 70 80 100				95

Variance σ^2

$$\sigma^2 = \frac{(X_1 - \mu)^2 + (X_2 - \mu)^2 + \dots + (X_N - \mu)^2}{N} = \frac{1}{N} \sum_{i=1}^N (X_i - \mu)^2$$



X	$X - \mu$	$(X - \mu)^2$
1	-4	16
3	-2	4
5	0	0
7	2	4
9	4	16
	$\Sigma = 0$	$\Sigma = 40$

var

$$\sigma^2 = \frac{40}{5} = 8$$

$$\sigma = \sqrt{\sigma^2} = 2.83$$

(std. deviation)

	μ	σ^2	σ
A	5	8	2.83
B	5	1.6	1.26
C	5	0	0

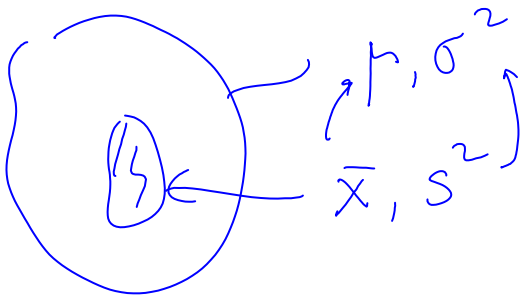
X	X- μ	(X- μ) ²
1	-4	16
3	-2	4
5	0	0
7	2	4
9	4	16
	$\Sigma = 0$	$\Sigma = 40$

$$\sigma^2 = \frac{40}{5} = 8$$

$$\sigma = \sqrt{\sigma^2} = 2.83$$

(std. deviation)

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	\$	\$ ²	\$



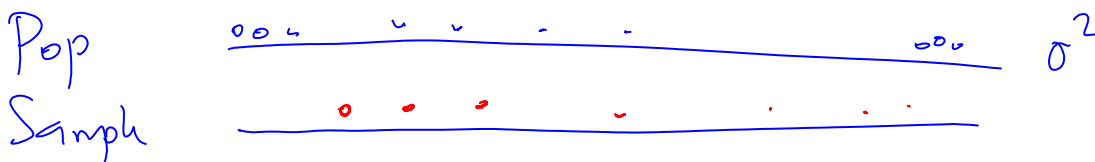
$$\mu = \frac{1}{N} \sum x_i$$

$$\bar{x} = \frac{1}{n} \sum x_i$$

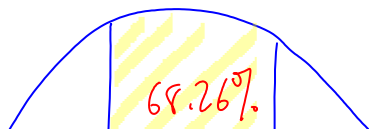
$$\sigma^2 = \frac{1}{N} \sum (x_i - \mu)^2$$

Sample var

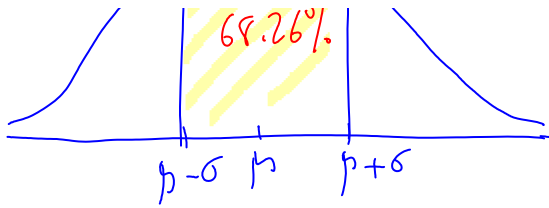
$$s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$$



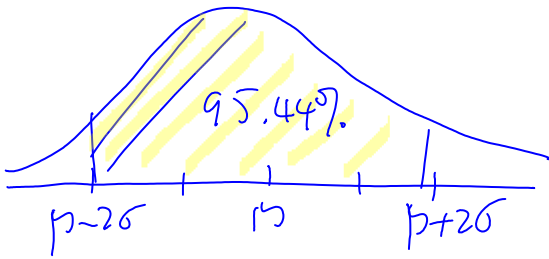
Empirical rule for "normal" pop's μ, σ



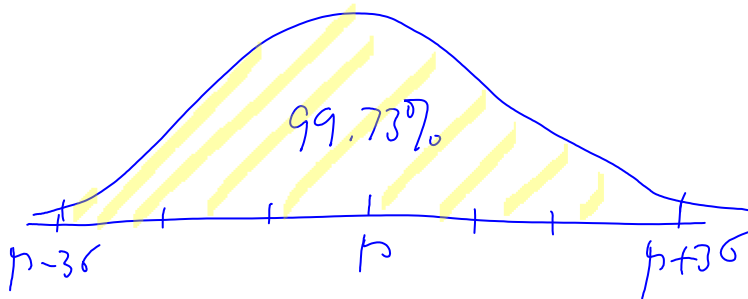
Tolerance interval
 $(\mu - \sigma, \mu + \sigma)$



$$(\mu \pm \sigma) = (\mu - \sigma, \mu + \sigma)$$



$$(\mu \pm 2\sigma)$$



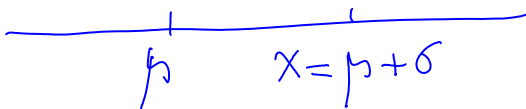
$$(\mu \pm 3\sigma)$$

Six-sigma (6σ)

99.9997%

(3 defects out of 1 million)

What z-scores?



$$\begin{aligned} x &= \mu + \sigma \\ x - \mu &= \sigma \\ \frac{x - \mu}{\sigma} &= \textcircled{1} \end{aligned}$$



$$\begin{aligned} x &= \mu + 2\sigma \\ \frac{x - \mu}{\sigma} &= \textcircled{2} \end{aligned}$$