(Response to Melody's question) 11/11/23

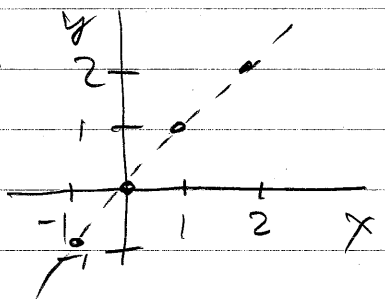
- When do we have a Correlation coefficient that equals one?

- This happens in the unusual case of a perfect "relationship" between x and y.

- In that case, the covariance turns into the variance and the std. dev. of Y becomes the std. dev. of X so that $S_x S_y = S_x S_x = S_x^2$ which gives

$$r = \frac{S_{xy}}{S_x S_y} = \frac{S_{xx}}{S_x S_x} = \frac{S_x^2}{S_x^2} = 1.$$

- Here's an example.



Here we have $y = x$ since all the points are on the same line.

So, $y_i = x_i$ and $\bar{y} = \bar{x}$ (right?)

This gives $S_{xy} = \frac{1}{n-1}\sum (x_i - \bar{x})(y_i - \bar{y}) = \frac{1}{n-1}\sum (x_i - \bar{x})^2 = S_x^2$

But also, $S_y^2 = \frac{1}{n-1}\sum (y_i - \bar{y})^2 = \frac{1}{n-1}\sum (x_i - \bar{x})^2 = S_x^2$

So, as above

$$r = \frac{S_{xy}}{S_x S_y} = \frac{S_x^2}{S_x S_x} = \frac{S_x^2}{S_x^2} = 1$$

The result also holds when $y = b_0 + b_1 x$ for any b_0, b_1