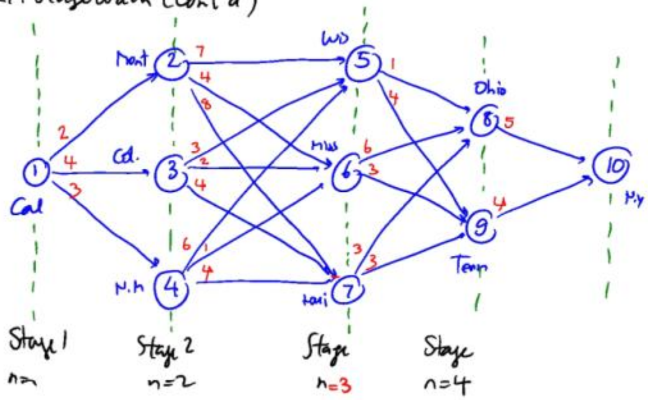
2011-12-05-EC01

Monday, December 05, 2011
11:11 AM

parlar-0711

Ex. Stagecoach (Cont'd)



(i) Myopic rule: Not good

(ii) Brute force : inefficient

(iii) D.P. approach (Bellman) ; Principle of Optimality

Some notation

s_n: state we are in currently in stage n

x_n. immediate destination (decision)

$c_n(s_n, x_n)$: immediate cost

$f_n(s_n, x_n)$. total cost from s_n to end

x_n^* : optimal decision in n

$f_n^*(s_n)$: min cost from s_n to end by making optimal decisions

Start at stage 4 (1 to go)

C01 Page 1

$$10 \times 10 \times 10 \times 10 \times 10 \times 10 = 10^6$$

$$20^{50} \cong 1.25 \times 10^{65}$$

(iii) DP approach

Principle of optimality : Given the current state we are in now, the remaining decisions must constitute an optimal policy leaving that state

Some notation

n: stage

s_n: state we are in currently in stage n

x_n: immediate destination in stage n

$c_n(s_n, x_n)$: cost of departing s_n by choosing x_n

$f_n(s_n, x_n)$: cost from stage n to end given we are in s_n and choose x_n

x_n^*: optimal decision in n

$f_n^*(s_n)$: min cost from n to end given we are in state s_n and we always decide optimally

Start at stage 4

From \ To	Cost to End: 10	Best decision	Min. Cost to End
8	5^*	10	5
9	4^*	10	4

s_n \ x_n	$f_4(s_4, x_4) = c_4(s_4, x_4)$: 10	x_4^*	$f_4^*(s_4)$
8	5^*	10	5
9	4^*	10	4

Stage 3

s_3 \ x_3	$f_3(s_3,x_3) = c(s_3,x_3) + f_4^*(s_4)$: 8	9	x_3^*	$f_3^*(s_3)$
5	$1+5=6^*$	$4+4=8$	8	6
6	$6+5=11$	$3+4=7^*$	9	7
7	$3+5=8$	$3+4=7^*$	9	7

Stage 2

$f_2(s_2,x_2) = c(s_2,x_2) + f_3^*(s_3)$

$s_2 \backslash x_2$	5	6	7	x_2^*	$f_2^*(s_2)$
2	7+6=13	4+7=11*	8+7=15	6	11
3	3+6=9*	2+7=9*	4+7=11	5/6	9
4	6+6=12	1+7=8*	4+7=11	6	8

Stage 1 $f_1(s_1,x_1) = c(s_1,x_1) + f_2^*(s_2)$

$s_1 \backslash x_1$	2	3	4	x_1^*	$f_1^*(s_1)$
1	2+11=13	4+9=13	3+8=11*	4	11

$$1 \xrightarrow{3} 4 \xrightarrow{1} 6 \xrightarrow{3} 9 \xrightarrow{4} 10 \quad : TC = 11$$

$$f_n^*(s_n) = \min_{\text{(all allowable decision)}} \{\text{cost in stage } n + f_{n+1}^*(\text{new state in } n+1)\}$$

$$f_n^*(s_n) = \min_{x_n} \{c_n(s_n,x_n) + f_{n+1}^*(s_{n+1})\}$$

2. Production/Inventory Problem

Expensive and low volume product

Sales contract signed

Data: Demand $d_1 = 2$, $d_2 = 2$, $d_3 = 3$, $d_4 = 3$

Prod. cap: $K = 5$

Storage ": $S = 3$

Can meet demand from inv. and/or production

n: quarter (period) d_n: demand in n

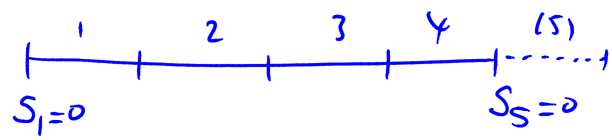
x_n: # produced in n

s_n: inv. at start of n

$$s_{n+1} = s_n + x_n - d_n$$

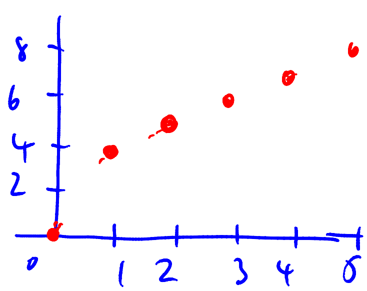
$s_1 = 0$ initial inv

$s_5 = 0$ end "



Cost: $h_n = 2$ holding cost/unit at end of n

Prod. cost: $$C_n(x_n) = \begin{cases} 3 + 1 \cdot x_n, & x_n = 1, 2, \ldots, 5 \\ 0 & x_n = 0 \end{cases}$$



$f_n^*(s_n)$: min cost of meeting demand for quarters $n, n+1, \ldots, 4$ if s_n units at start of n

$$s_{n+1} = s_n + x_n - d_n \leq S = 3$$

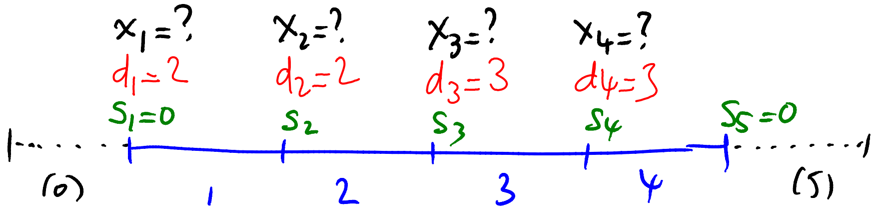
$$s_{n+1} = s_n + x_n - d_n \geq 0$$

$$0 \leq x_n \leq K = 5$$

s_{n+1}

$$f_n^*(s_n) = \min_{\substack{0 \le x_n \le K \\ 0 \le s_n + x_n - d_n \le S}} \left\{ \underbrace{c_n(x_n) + h_n(s_n + x_n - d_n)}_{\text{current cost}} + \underbrace{f_{n+1}^*(s_n + x_n - d_n)}_{\text{min cost for future}} \right\}$$

$$f_5^*(s_5) = 0$$



Q4 $d_4 = 3$

$c_4(x_4) + h_4(s_4 + x_4 - d_4) + f_5^*(s_5)$

s_4 \ x_4	0	1	2	3	4	5	x_4^*	$f_4^*(s_4)$
0				$6+0+0=6^*$			3	6
1			$5+0+0=5^*$				2	5
2		$4+0+0=4^*$					1	4
3	$0+0+0=0^*$						0	0

Q3 $d_3 = 3$

$c_3(x_3) + h_3(s_3 + x_3 - d_3) + f_4^*(s_4^*)$

s_3 \ x_3	0	1	2	3	4	5	x_3^*	$f_3^*(s_3)$
0				$6+0+6=12^*$	$7+2+5=14$	$8+4+4=16$	3	12
1			$=11^*$	$=13$	$=15$	$=14$	2	11
2		$=10^*$	$=12$	$=14$	$=13$		1	10
3	$=6^*$	$=11$	$=13$	$=12$			0	6

Q2 $(d_2 = 2)$

s_2 \ x_2	0	1	2	3	4	5	x_2^*	$f_2^*(s_2)$
0			$=17^*$	$=19$	$=21$	$=20$	2	17
1		$=16^*$	$=18$	$=20$	$=19$		1	16
2	$=12^*$	$=18$	$=19$	$=18$			0	12
3	$=13^*$	$=18$	$=17$				0	13

Q1 $(d_1 = 2)$

$s_1 \backslash x_1$	0	1	2	3	4	5	x_1^*	$f_1^*(s_1)$
0			$5+0+17=22^*$	$=24$	$=23$	$=27$	2	22
1		$=21^*$	$=23$	$=22$	$=26$		1	21

Sensitivity analysis

Min cost = \$22

3. Knapsack

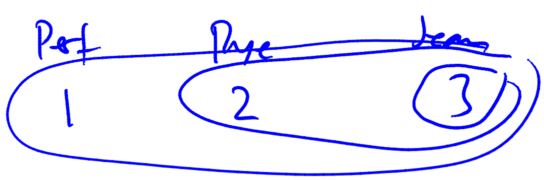
Knapsack cap = 7 lb

Item	Lbs	Profit
1. Perfume	1	\$70
2. Rye	2	\$90
3. Jeans	3	120

$$\text{Max } z = 70x_1 + 90x_2 + 120x_3$$

$$\text{s.t. } x_1 + 2x_2 + 3x_3 \le 7$$

$$x_1, x_2, x_3 \ge 0 \text{ integers}$$



x_n: # of item n to include
s_n: # lbs left (available) in knapsack before item n is included

$f_n^*(s_n)$: max profit $n, n+1, .., 3$

$f_n^*(s_n)$: max profit $n, n+1, .., 3$

Stage 3 Jeans (3 lbs)

$s_3 \backslash x_3$	0	1	2	x_3^*	$f_3^*(s_3)$	Cap. left
0	0	–	–	0	0	0
1	0	–	–	0	0	1
2	0	–	–	0	0	2
3	0	120	–	1	120	0
4	0	120	–	1	120	1
5	0	120	–	1	120	2
6	0	120	240	2	240	0
7	0	120	240	2	240	1

Stage 2 Rye 2lbs

$s_2 \backslash x_2$	0	1	2	3	x_2^*	$f_2^*(s_2)$
0	=0				0	0
1	=0				0	0
2	=0				1	90
3	=120				0	120
4					2	180
5					1	210
6					3	270
7					2	300

Stage 1 Perfume 1 lb

$s_1 \backslash x_1$	0	7	x_1^*	$f_1^*(s_1)$
7			7	490