Interfaces Journal

c) Decision Making with experimentation

Currently. $Pr(S_1) = .4$, $Pr(S_2) = .6$ (prior probs)

Sunspotz considers hiring a consulting firm
Econometrics in Action (EIA)

EIA: correctly predict Hi demand 80% time
" " normal " 70% "

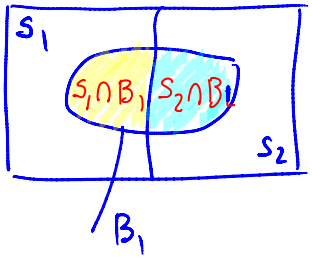
Predictions: B_1: predict Hi demand
B_2, " mod. "

Likelihoods: $Pr(B_1|S_1) = .6$, $Pr(B_2|S_1) = .2$
$Pr(B_1|S_2) = .3$, $Pr(B_2|S_2) = .7$

How to find $Pr(S_1|B_1) = ?$
$Pr(S_2|B_2) = ?$

$Pr(B_1|S_1) = \frac{Pr(B_1 \cap S_1)}{Pr(S_1)}$

$$Pr(S_1|B_1) = \frac{Pr(S_1 \cap B_1)}{Pr(B_1)} = \frac{Pr(S_1 \cap B_1)}{Pr(S_1 \cap B_1) + Pr(S_2 \cap B_1)}$$



$$Pr(S_1 \cap B_1) = Pr(B_1|S_1)\,Pr(S_1)$$
$$Pr(S_2 \cap B_1) = Pr(B_1|S_2)\,Pr(S_2)$$

$$\therefore Pr(S_1|B_1) = \frac{Pr(B_1|S_1)\,Pr(S_1)}{Pr(B_1|S_1)\,Pr(S_1) + Pr(B_1|S_2)\,Pr(S_2)}$$ Bayes's formula

Tabular calculations (given B_1, say)

	Prior	Likelihoods	Joint	Posterior

State	$Pr(S_i)$	$Pr(B_1 \mid S_i)$	$Pr(B_1 \cap S_i)$	$Pr(S_i \mid B_1)$
S_1	.4	.8	.32	.32/.50 = .64
S_2	.6	.3	.18	.18/.50 = .36
			$Pr(B_1) = .50$	1.00

←

Given B_2

		$Pr(B_2 \mid S_i)$		$Pr(S_i \mid B_2)$
S_1	.4	.2	.08	.08/.50 = .16
S_2	.6	.7	.42	= .84
			$Pr(B_2) = .50$	1.00

What to do?

	S_1	S_2
A_1	5	-1
A_2	3	0.5

If B_1:

$E(A_1) = .64(5) + .36(-1) = 2.84$ ←

$E(A_2) = .64(3) + .36(0.5) = 2.10$

If B_2:

$E(A_1) = .16(5) + .84(-1) = -.04$

$E(A_2) = .16(3) + .84(.5) = .90$ ←

Info. cost $100,000 ($0.1M) to use EVA

	Decision	No cost $E(A^*)$	Cost $E(A^*)$
B_1	A_1	2.84	2.84 − .1 = 2.74
B_2	A_2	.90	.90 − .1 = .80

Q: Is experimentation worth it?

Value of Experimentation

① Expected Value of Perfect Info (EVPI)

A clairvoyante can tell us exactly what will happen

Info: →	S_1 ↓	S_2 ↓
A_1	(5)	-1
A_2	3	(0.5)

Max	5	0.5
Prob	.4	.6

Exp. payoff with Perfect Info

$= EPPI = .4(5) + .6(.5) = 2.3$

But our ⟶ $EMV = 1.5$

$EVPI = EPPI - EMV$

$= 2.3 - 1.5 = 0.8 = \$800{,}000$

② Expected Value of Sample Info (EVSI) ①
" " experimentation (EVE)

Expected payoff with experimentation (EPE)

(1) Prediction	(2) Prob	(3) Opt. Dec	(4) Opt. EMV | B_i	(5) = (2)×(4)
B_1	.5	A_1	2.84	1.42
B_2	.5	A_2	0.90	.45
				EPE = 1.87

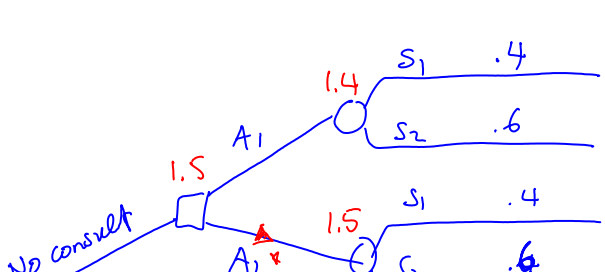
$EVE = EVSI = EPE - EMV = 1.87 - 1.50 =$

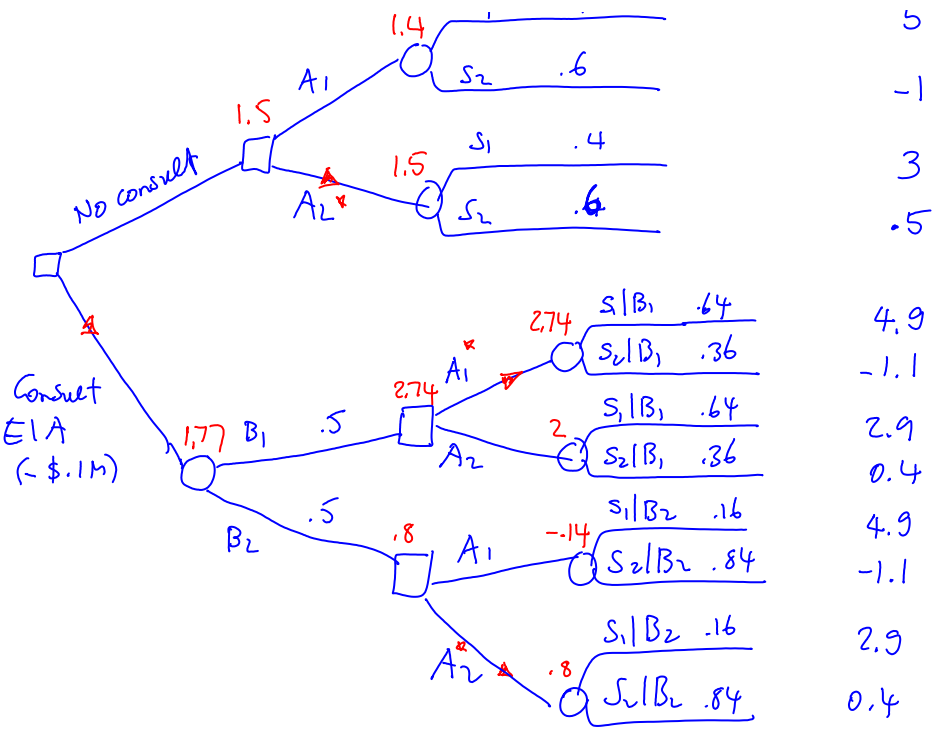
$= .37$ ($370,000)

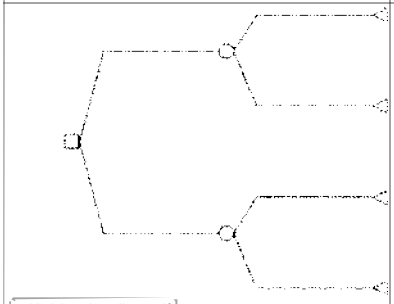
Can pay up to $370,000 to EIA

d) Decision Trees (TreePlan)

□ : decision node
○ : chance "





Sunspotz Decision Tree - No experiment																		
TreePlan Academic Version							0.4			**Only For Academic Use**		I put this table here just to copy the prior probabilities.						
 I added the color cells myself							High (S1)											
										5		**Data:**		P(Predict B | State S)				
			Expand (A1)				10	5				State of Nature	Prior	Predicting				
												Nature (Demand)	Probability	B1	B2			
			-5	1.4			0.6					High (S1)	0.4	0.8	0.2			
							Normal (S2)					Normal (S2)	0.6	0.3	0.7			
										-1								
							4	-1										
	2																	
1.5							0.4											
							High (S1)					**Posterior**		P(State S | Predict B)				
										3		**Probabilities:**		State of Nature				
			Modernize (A2)				5	3				Predicting	P(Finding)	High (S1)	Normal (S2)			
												B1	0.5	0.64	0.36			
			-2	1.5			0.6					B2	0.5	0.16	0.84			
							Normal (S2)											
										0.5								
							2.5	0.5										

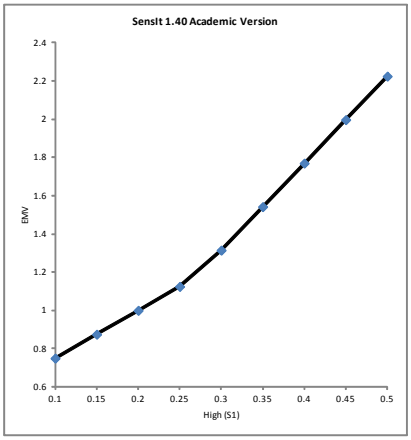
Analysis Using SensIt

One input → one output

or many " → " "

Ex. Finance

$$F = P(1+i)^n$$



SensIt for

Pr(S1) ⇒ .10 (.05) .50