Bus Q771: STOCHASTIC PROCESSES WITH BUSINESS APPLICATIONS
Fall 2013 Course Outline

Dr. Mahmut Parlar
< Operations Management Area >

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1 COURSE OBJECTIVE

Stochastic (random) processes involves the study of systems which evolve randomly over time. The main objective of the course is an understanding of the behaviour of these systems, especially in the long run. Once this "descriptive" stage of the analysis is completed, we pass on to the "normative" stage where we attempt to control the behaviour of the system to improve its performance. This Ph.D.-level course surveys the important tools of stochastic processes to model and solve business problems arising in inventory management, waiting line systems and others.

The students are expected to have a basic understanding of probability theory at a level equivalent to Ross's "A First Course in Probability," [17] [nth Edition, $n \geq 1$].

The book by Grinstead and Snell [8] ("Introduction to Probability") <`http://www.math.dartmouth.edu/~prob/prob/prob.pdf`> is available online and it should also be consulted for reference on probability review.

Appendix A in Heyman and Sobel [8] should also be consulted for background material in probability theory.

2 INSTRUCTOR AND CONTACT INFORMATION

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`http://profs.degroote.mcmaster.ca/ads/parlar/index.html`

3 CLASS TIME and ROOM

- Mondays from 12:30–3:30 p.m. in DSB-421, starting September 9, 2013.
  - We will move the September 23, 2013 meeting to the evening (as I have to attend an important meeting in the afternoon).

4 COURSE DESCRIPTION

The course covers stochastic processes and their applications in modeling the business environment evolving over time. After a brief review of probability theory, we discuss the Poisson/exponential duality, renewal theory, discrete and continuous-time Markov chains and, time permitting, Brownian motion. Individual student participation is also required in selecting, researching and presenting topics on stochastic process applications.

- Prerequisite: Enrolment in the PhD Program or permission of the instructor.

5 COURSE WEBSITE

`http://profs.degroote.mcmaster.ca/ads/parlar/courses/Q771/`

6 LEARNING OUTCOMES

Upon completion of this course, students will be able to complete the following key tasks:

- formulate and solve probabilistic problems using random variables,
- distinguish between Poisson process and the exponential random variable and apply this knowledge to solve problems involving memoryless processes,
- use renewal theory to solve problems where Poisson is not a realistic process,
- use Markov chains in discrete- and continuous-time to solve inventory and queueing problems.

7 REQUIRED COURSE MATERIALS AND READINGS

7.1 Main References

The course will be based on instructor's lecture notes. However, we will make frequent use of the following books **both which will be placed in Innis Reserve**.

- E. P. C. Kao [9] (rich in examples),
- M. Parlar [14] (the use of Maple in solving non-trivial OR problems including those in stochastic processes),
- S. M. Ross [18] (mainly theorems).

7.2 Software Use

- We will make frequent (and substantial) use of the computer algebra system Maple (`maplesoft.com`) in this course using Parlar [14].
- See the next link for information and downloads on this book and related software.

`http://profs.degroote.mcmaster.ca/ads/parlar/ORMapleBook/ORMapleBook.html`

7.3 Suggested Reading

There is a large number of excellent books on stochastic processes some of which are listed below.

- D. J. Bartholomew [1], U. N. Bhat [2], D. R. Cox [3], D. R. Cox and H.D. Miller [4], R. Goodman [5], W. Grassman [6], C. M. Grinstead and J. L. Snell [7], D. Heyman and M. Sobel [8], E. P. C. Kao [9], S. Karlin and H. M. Taylor [10], J. Kemeny and J. Snell [11], L. Kleinrock [12], J. Medhi [13], M. Parlar [14], E. Parzen [15], S. M. Ross [16], S. M. Ross [18], S. M. Ross [19], L. Takács [20], H. M. Taylor and S. Karlin [21], H. C. Tijms [22], H. C. Tijms [23].

8 EVALUATION

8.1 Allocation of Marks

2 Assignments	20%
Midterm Examination	30%
Survey Paper	10%
Final Examination	40%

8.2 Grade Conversion

Grade (Points)	**Percent**
A+ (12)	90–100
A (11)	85–89
A− (10)	80–84
B+ (9)	75–79
B (8)	70–74
B− (7)	60–69
F (0)	0–59

8.3 Literature Survey

Each student is responsible for providing a literature survey on a particular topic of his/her choice in stochastic processes. Some suggested topics are

- Applications of renewal theory,
- Applications of discrete-time Markov chains,
- Applications of continuous-time Markov chains,
- Applications of stochastic processes in
  - Finance, or
  - Marketing, or
  - Production/Operations Management, or
  - Accounting, or
  - Manpower Planning.

Due date for this literature survey report (type-written—LaTeX preferred—maximum 10 double-spaced pages) is **December 13, 2013 (Friday, 12 noon).** Students should get the approval of the instructor on the topic chosen before embarking on this project.

9 ACADEMIC DISHONESTY

It is the student's responsibility to understand what constitutes academic dishonesty. Please refer to the University Senate Academic Integrity Policy at the following URL:

`http://www.mcmaster.ca/policy/Students-AcademicStudies/AcademicIntegrity.pdf`

This policy describes the responsibilities, procedures, and guidelines for students and faculty should a case of academic dishonesty arise. Academic dishonesty is defined as to knowingly act or fail to act in a way that results or could result in unearned academic credit or advantage. Please refer to the policy for a list of examples. The policy also provides faculty with procedures to follow in cases of academic dishonesty as well as general guidelines for penalties. For further information related to the policy, please refer to the Office of Academic Integrity at:

`http://www.mcmaster.ca/academicintegrity`

10 MISSED ACADEMIC WORK

Late assignments will not be accepted. No extensions are available except under extraordinary circumstances. Please discuss any extenuating situation with your instructor at the earliest possible opportunity.

11 POTENTIAL MODIFICATIONS TO THE COURSE

The instructor and university reserve the right to modify elements of the course during the term. The university may change the dates and deadlines for any or all courses in extreme circumstances. If either type of modification becomes necessary, reasonable notice and communication with the students will be given with explanation and the opportunity to comment on changes. It is the responsibility of the student to check their McMaster email and course websites weekly during the term and to note any changes.

12 COPYRIGHT

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`http://www.copyright.mcmaster.ca/Access_Copyright_Agreement`

13 STUDENT ACCESSIBILITY SERVICES

Student Accessibility Services (SAS) offers various support services for students with disabilities. Students are required to inform SAS of accommodation needs for course work at the outset of term. Students must forward a copy of such SAS accommodation to the instructor normally, within the first three (3) weeks of classes by setting up an appointment with the instructor. If a student with a disability chooses NOT to take advantage of an SAS accommodation and chooses to sit for a regular exam, a petition for relief may not be filed after the examination is complete. The SAS website is:

`http://sas.mcmaster.ca`

14 COURSE SCHEDULE

- **Review of Basic Probability Theory** (2 weeks)
  - Probability spaces. Random variables. Expected value. Generating functions and Laplace transforms. Conditional expectations.
- **Exponential Distribution and the Poisson Process** (2 weeks)
  - Exponential distribution. Lack of memory. Hazard rate function. Erlang distribution. Poisson process. Conditional distribution of arrival times. Nonhomogeneous Poisson processes. Compound Poisson processes. Applications.
- **Renewal Theory** (3 weeks)
  - Renewal process. Renewal equation. Limit theorems. Alternating renewal process. Backward and forward recurrence times. Inspection paradox. Renewal reward processes. Applications.
- **Discrete-time Markov Chains** (3 weeks)
  - Definitions. Chapman-Kolmogorov equations. Transient solutions. Classification of states. The fundamental matrix. Semi-Markov processes. Applications.
- **Continuous-time Markov Chains** (2 weeks)
  - Definitions. Birth-and-death processes. Chapman-Kolmogorov equations. Limiting probabilities. Time reversibility. Uniformization. PASTA ("Poisson Arrivals See Time Averages") property. Applications.
- **Brownian Motion** (1 week)
  - Arithmetic and geometric Brownian motion processes (1 week).

References

[1] D. J. Bartholomew. *Stochastic Models for Social Processes.* John Wiley and Sons, Chichester, 3rd edition, 1982.

[2] U. N. Bhat. *Elements of Applied Stochastic Processes.* John Wiley, New York, 2nd edition, 1984.

[3] D. R. Cox. *Renewal Theory.* Methuen, London, 1962.

[4] D. R. Cox and H. D. Miller. *The Theory of Stochastic Processes.* Chapman and Hall, London, 1965.

[5] R. Goodman. *Introduction to Stochastic Models.* Benjamin/Cummings, Menlo Park, 1988.

[6] W. Grassman. *Stochastic Systems for Management.* North Holland, 1981.

[7] C. M. Grinstead and J. L. Snell. *Introduction to Probability.* American Mathematical Society, Providence, RI, 2nd edition, 1997. This book is freely redistributable under the terms of the GNU Free Documentation License from `http://www.math.dartmouth.edu/~prob/prob/prob.pdf`.

[8] D. P. Heyman and M. J. Sobel. *Stochastic Models in Operations Research, Volume I: Stochastic Processes and Operating Characteristics.* McGraw-Hill, New York, 1982.

[9] E. P. C. Kao. *An Introduction to Stochastic Processes.* Duxbury, Belmont, California, 1997.

[10] S. Karlin and H. M. Taylor. *A First Course in Stochastic Processes.* Academic Press, San Diego, 2nd edition, 1975.

[11] J. G. Kemeny and J. L. Snell. *Finite Markov Chains.* Springer-Verlag, New York, 1976.

[12] L. Kleinrock. *Queueing Systems, Volume 1: Theory.* John Wiley, New York, 1975.

[13] J. Medhi. *Stochastic Processes.* John Wiley, New York, 1994.

[14] M. Parlar. *Interactive Operations Research with Maple: Methods and Models.* Birkhäuser, Boston, 2000.

[15] E. Parzen. *Stochastic Processes.* Holden-Day, 1962.

[16] S. Ross. *Applied Probability Models with Optimization Applications.* Holden Day, San Francisco, Calif., 1970.

[17] S. Ross. *A First Course in Probability.* Macmillan, New York, 1976.

[18] S. Ross. *Stochastic Processes.* John Wiley, New York, 1983.

[19] S. Ross. *Introduction to Probability Models.* Academic Press, Orlando, Fla., 3rd edition, 1985.

[20] L. Takács. *Stochastic Processes.* Methuen, London, 1966.

[21] H. M. Taylor and S. Karlin. *An Introduction to Stochastic Modeling.* Academic Press, Inc., San Diego, CA, 1984.

[22] H. C. Tijms. *Stochastic Modeling and Analysis*. John Wiley, Chichester, 1986.

[23] H. C. Tijms. *Stochastic Models: An Algorithmic Approach*. John Wiley, Chichester, 1994.

SEPTEMBER 2013