DO ↓

12. How to tell if A or S is exponential

So far we usually assumed that the interarrival and the service times were exponential. It is important to be able to check statistically whether this assumption can be justified

Ex. length of telephone calls in a travel agency

pr. 8, 12.1
p. 370

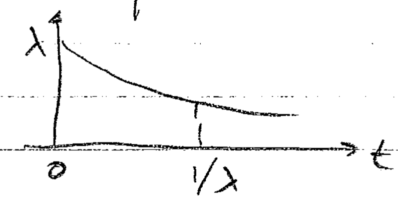
Suppose we have the following sample (in seconds): $(t_1, \ldots, t_{24})$

4, 6, 5, 8, 9 / 10, 12, 8, 16, 20 / 24, 27, 33, 37, 43 / 50, 58, 68, 70, 78 / 88, 100, 120, 130.

Do these data indicate that length was exponential?

Exponential : $f(t) = \lambda e^{-\lambda t}, \ t \geq 0$

$E(T) = 1/\lambda$

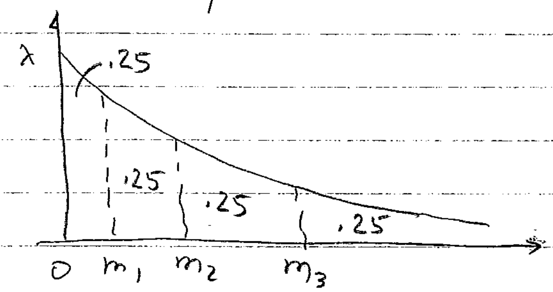


Estimate of average time : $\frac{1}{\hat{\lambda}} = \frac{t_1 + \cdots + t_{24}}{24}$

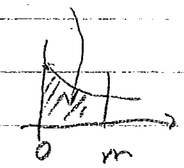
$= \frac{1024}{24} = 42.66$ sec.

Hence $\hat{\lambda} = 1/42.66 = .0234$

Suppose we use 4 categories so that each category has 6 observations. (expected)



$Pr(S \leq m) = 1 - e^{-\lambda m}$



$P(S \leq m_1) = 1 - e^{-.0234 m_1} = .25 \Rightarrow m_1 = 12.3$

$P(S \leq m_2) = 1 - e^{-.0234 m_2} = .50 \Rightarrow m_2 = 29.6$

$P(S \leq m_3) = 1 - e^{-.0234 m_3} = .75 \Rightarrow m_3 = 59.2$

In general, if $1 - e^{-\lambda m} = p$, then $m = -\ln(1-p)/\lambda$

Category	Range	Observed	Expected
1	0 - 12.3	8	6 (= 24 × .25)
2	12.4 - 29.6	4	6
3	29.7 - 59.2	5	6
4	59.3 -	7	6

$$\chi^2(obs) = \sum_{i=1}^{4} \frac{(O_i - e_i)^2}{e_i}, \ .01$$

$$\chi^2(obs) = \frac{(8-6)^2}{6} + \frac{(4-6)^2}{6} + \frac{(5-6)^2}{6} + \frac{(7-6)^2}{6} = 1.67$$

H_0: $t_1, \dots, t_{44}$ are from exp

H_a: " " not "

If $\chi^2(obs)$'s very large, we'll reject the hypothesis that our observations come from exponential.

If $\chi^2(obs) > \chi^2_{k-r-1}(\alpha)$,

critical value

$\alpha = Pr$(Type I error)
$= Pr$(reject exp. assumption when it's true)

find innocent guilty

k : # categories
r : # parameters ,

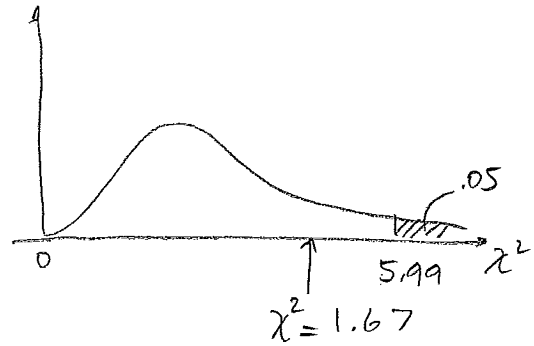
exp. dist has one par.

then $k=4$, $r=1$, and use $\alpha = .05$

$$\chi^2_{k-r-1}(\alpha) = \chi^2_{4-1-1}(.05) = \chi^2_2(.05) = 5.99 \text{ (Table 9, pg. 374)}$$

= CHIINV(alpha, k-r-1)
= CHIINV(.05, 2)

(Text has a typo: CHIIN



∴ Accept hypothesis that observations are exponential.

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k may be determined by Sturgs' Rule

n: # observat

k = NCEL = 1 + 3.3 · LOG$_{10}$(n) = 1 + 4.55 = 5.55

HWID = $\dfrac{X_{max} - X_{min}}{NCEL}$ = 22.5

H_0 : innocent
H_a : guilty

	Innoc H_0	Guilty H_a
Find innoc	correct	Type II
" guilty	Type I	correct