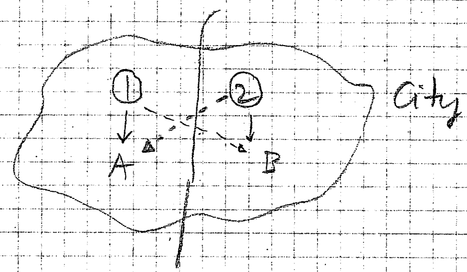
Ex. Response Areas for Two Emergency Units

Two units (e.g., ambulances) $i = 1, 2$

Two regions $j = A, B$



1 for A, 2 for B } units may serve the other region if only one unit available

When both busy, arriving call lost

Arrival from j $(= A, B)$ Poisson with λ_j

Service exponential with μ_{ij} $(i = 1, 2;\ j = A, B)$

μ_{ij}: service rate when region j call is served by i

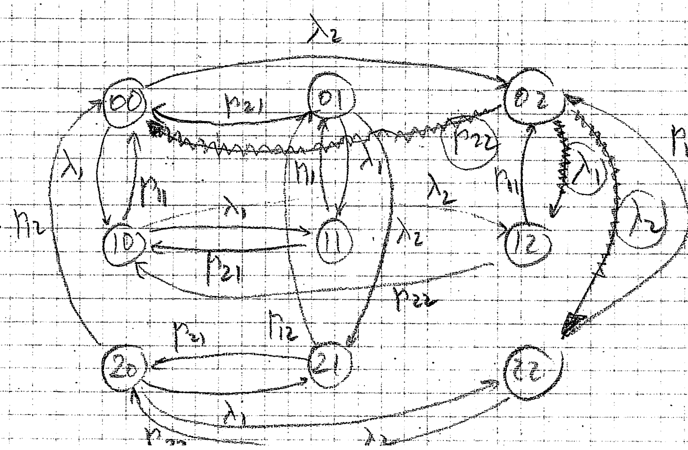
$X_i(t)$: Status of unit i at t → $= 0$ idle, $= 1$ serving A, $= 2$ " B

Bivariate stochastic process $X(t) = (X_1(t), X_2(t))$

$X_1 \backslash X_2$	0	1	2
0	00	01	02
1	10	11	12
2	20	21	22

} State space

Interpret
02 → 00
12
22



$$
Q = \begin{array}{c|ccccccccc}
 & 00 & 01 & 02 & 10 & 11 & 12 & 20 & 21 & 22 \\
\hline
00 & * & & \lambda_2 & \lambda_1 & & & & & \\
01 & p_{21} & * & & & \lambda_1 & & & \lambda_2 & \\
02 & p_{22} & & * & & & \lambda_1 & & & \lambda_2 \\
10 & p_{11} & & & * & \lambda_1 & \lambda_2 & & & \\
11 & & p_{11} & & p_{21} & * & & & & \\
12 & & & p_{11} & p_{22} & & * & & & \\
20 & p_{12} & & & & & & * & \lambda_1 & \lambda_2 \\
21 & & p_{12} & & & & & p_{21} & * & \\
22 & & & p_{12} & & & & p_{22} & & *
\end{array}
$$

$*$: $-$(sum of the same row elements)

let $\lambda_1 = .1$ $\quad p_{11} = .5$ $\quad p_{12} = .25$ $\quad$ (rate per hr)

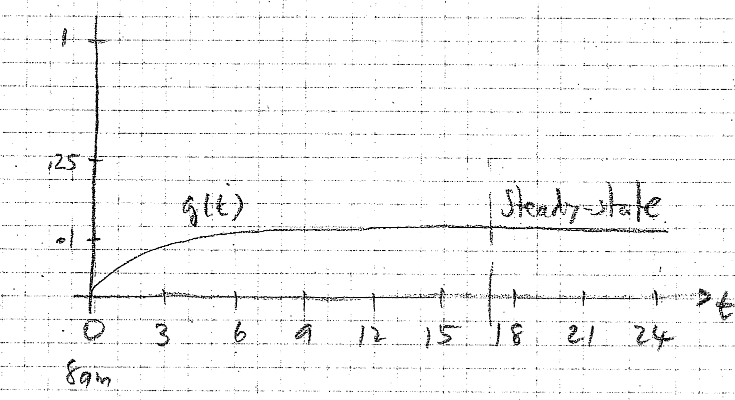
$\lambda_2 = .125$ $\quad p_{21} = .2$ $\quad p_{22} = .4$

Nondesignated units take longer

let $p_{ij}(t) = P\{X_1(t) = i, X_2(t) = j\}$

$$g(t) = \sum_{i=1}^{2} \sum_{j=1}^{2} p_{ij}(t) \; : \; Pr(\text{incoming call lost})$$

Solving using a DE soln method



Decision Problem
- How many units @ each location?
- How many locations?