# Game-Theoretic Models in Supply Chain Management

2012 PhD Summer Academy

Zaragoza Logistics Center, SPAIN

Professor Mahmut Parlar, McMaster University

**ASSIGNMENT #2**

- **Due date: June 29, 2012 (Friday), 10:15 a.m., in class.**
- **Each question is assigned 25 marks.**

1. Refer to Section 2.2.2 of the paper by Wu and Parlar [1]. Assume that $[a, b \mid s_1, s_2 \mid c_1, c_2] = [0.9, 0.9 \mid 15, 15 \mid 8, 8]$ and that demand densities are exponential, i.e., $f(x) = \lambda e^{-\lambda x}$ and $h(y) = \mu e^{-\mu y}$ with respective parameters $(\lambda, \mu) = \left(\frac{1}{30}, \frac{1}{30}\right)$. Find the Stackelberg equilibrium for the game assuming that player 1 is the leader and player 2 is the follower.

2. Consider the Cournot game of incomplete information and assume now that both firms can have high costs or low costs. In particular, assume that the marginal cost of firm 2 is either high, $c_H$, or low, $c_L$, where $c_H > c_L \geq 0$. Firm 2 knows its marginal cost but firm 1 only knows that it is $c_H$ with probability $\theta$ or $c_L$ with probability $1 - \theta$. Assume now that also firm 1 can have high costs or low costs, say $c_h$ with probability $\pi$ and $c_\ell$ with probability $1 - \pi$ and $c_h > c_\ell \geq 0$.

   (a) Show that the conditions leading to the Nash equilibrium are,

$$
\begin{aligned}
q_\ell &= \frac{1}{2}[a - c_\ell - \theta q_H - (1-\theta) q_L] \\
q_h &= \frac{1}{2}[a - c_h - \theta q_H - (1-\theta) q_L] \\
q_L &= \frac{1}{2}[a - c_L - \pi q_h - (1-\pi) q_\ell] \\
q_H &= \frac{1}{2}[a - c_H - \pi q_h - (1-\pi) q_\ell].
\end{aligned}
$$

   (b) Use the folllowing data to compute the Nash equilibrium for this two-sided incomplete information game: $a = 10$, $(c_H, c_L) = (2, 1)$, $(c_h, c_\ell) = (3, 2)$, $\theta = 0.7$ and $\pi = 0.4$.

   (c) Can the equilibrium be found analytically? How?

3. Consider the simple continuous strategy game discussed in class with $f(x, y) = -2x^2 + 5xy$ as P1's payoff function and $g(x, y) = -3y^2 + 2xy + y$ as P2's payoff function where $x, y \geq 0$.

   (a) Find the Stackelberg equilibrium when P1 is the leader,

   (b) Find the Stackelberg equilibrium when P2 is the leader.

4. Consider the adverse selection mechanism design problem discussed in class with the following data: $S(q) = 20q - q^2/2$ and $\Theta = \{\theta_1, \theta_2\} = \{2, 4\}$.

   (a) Find the "first best" menu of contracts under the complete information setting,

   (b) Explain why the "first best" menu would not work without incentive feasibility constraints

   (c) Solve the Principal's optimization problem which includes the incentive feasibility constraints

## References


[1] H. Wu and M. Parlar. Games with incomplete information: A simplified exposition with inventory management applications. *International Journal of Production Economics*, 133:562–577, 2011.