Game-Theoretic Models in Supply Chain Management

2012 PhD Summer Academy

Zaragoza Logistics Center, SPAIN

Professor Mahmut Parlar, McMaster University

ASSIGNMENT #1

- **Due date: June 25, 2012 (Monday), 10:00 a.m., in class.**
- **Each question is assigned 20 marks.**

1. Consider the "**Rock, Paper and Scissors**" game discussed in class. Find the players' optimal strategies in this zero-sum game using linear programming.

2. Consider the "**Battle at the Bismarck Sea**" problem discussed in class. Show that the two players' linear programs are the duals of each other. Refer to your favourite text on operations research to recall what duality means.

3. Consider the "**Battle of the Sexes**" game discussed in class with the payoff matrix given as

$$\begin{pmatrix} 2,1 & 0,0 \\ 0,0 & 1,2 \end{pmatrix}.$$

   Find all three Nash equilibria using manual calculations and check your results by solving the problem with Gambit.

4. In a production problem with two firms, the profit functions for the two firms are given as

$$\begin{aligned} \text{Firm 1} &: \quad P_1(x,y) = 96x - 4x^2 - x^3 - 8xy \\ \text{Firm 2} &: \quad P_2(x,y) = 80y - 4y^2 - y^3 - 8xy \end{aligned}$$

   where x and y are the production quantities of firms 1 and 2, respectively.

   (a) Show using Debreu's existence theorem that the Nash equilibrium exists.

   (b) Show using the contraction mapping theorem that the equilibrium must be unique and find it.

5. Refer to the paper by Wu and Parlar [1]. Assume that $[a, b \mid s_1, s_2 \mid c_1, c_2] = [0.9, 0.9 \mid 15, 15 \mid 8, 8]$ and that demand densities are exponential, i.e., $f(x) = \lambda e^{-\lambda x}$ and $h(y) = \mu e^{-\mu y}$ with respective parameters $(\lambda, \mu) = \left(\frac{1}{30}, \frac{1}{30}\right)$.

   (a) Does the Nash equilibrium exist and is it unique? Explain.

   (b) If the Nash equilibrium exists, find it for this symmetric game.

References

[1] H. Wu and M. Parlar. Games with incomplete information: A simplified exposition with inventory management applications. *International Journal of Production Economics*, 133:562–577, 2011.