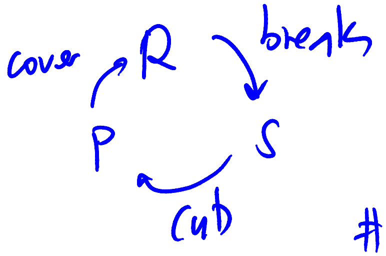
2012-06-21

Sunday, June 17, 2012
6:06 PM

0. What's Game Theory (GT)?

a) Simple example (zero-sum) #2

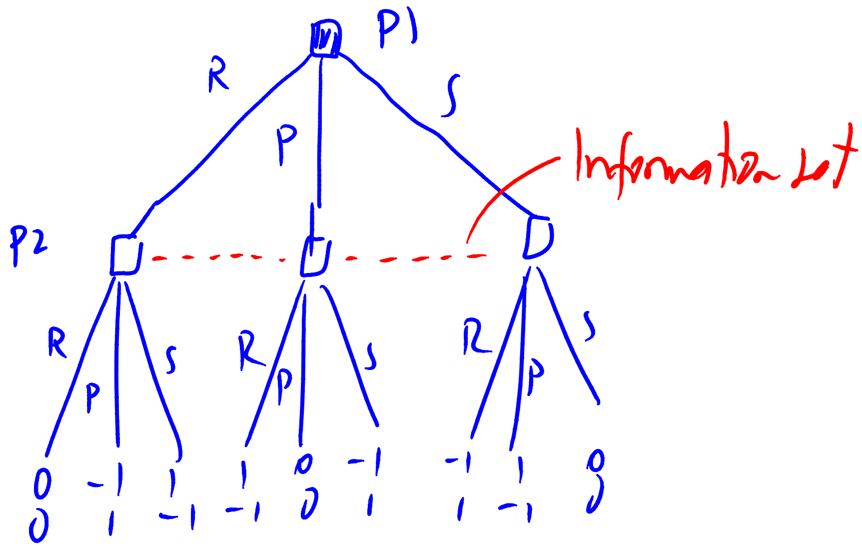


#1

	R	P	S
R	0,0	-1,1	1,-1
P	1,-1	0,0	-1,1
S	-1,1	1,-1	0,0

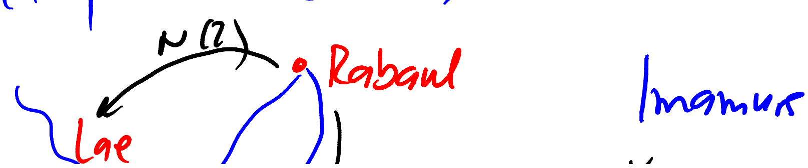
strategic form

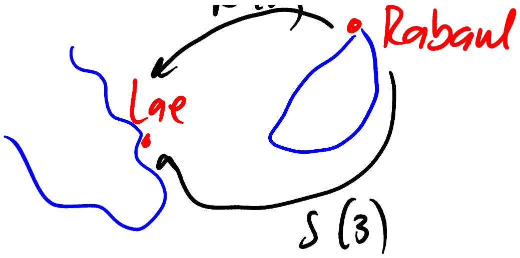
Extensive form



Another example: Battle of Bismarck Sea

1943 (Papua New Guinea)





Imamura

Kenney

$$
\begin{array}{cc}
 & \text{Imamura} \\
\text{Kenney} &
\begin{array}{c|cc}
 & N & S \\
\hline
N & (2,-2) & (2,-2) \\
S & (1,-1) & (3,-3)
\end{array}
\end{array}
$$

days of bombing

$$
\begin{array}{c|cc|c}
 & N & S & \text{Row min} \\
\hline
N & 2 & 2 & 2 \\
S & 1 & 3 & 1 \\
\hline
\text{Col. max} & 2 & 3 &
\end{array}
$$

← maxmin

Saddle pt

minmax

$v = 2$ (value of game)

Assignment #1 due Monday 10:00 am

LP approach

$$
\begin{array}{cc}
 & P2 \\
 & y_1 \quad y_2
\end{array}
$$

$$
\text{P1}\quad \begin{array}{c} x_1 \\ x_2 \end{array}\begin{pmatrix} 2 & 2 \\ 1 & 3 \end{pmatrix}
$$

$x_i = \Pr(\text{P1 plays } i)$, $i = 1, \dots, m$, $\sum x_i = 1$

$y_j = \Pr(\text{P2 } \text{''} \ j)$, $j = 1, \dots, n$ $\sum y_j = 1$

P1. max v

s.t.
$$2x_1 + x_2 \geq v$$
$$2x_1 + 3x_2 \geq v$$
$$x_1 + x_2 = 1$$
$$x_1, x_2 \geq 0, \quad v: \text{free}$$
$$x_1 = 1, \ x_2 = 0, \ v = 2$$

P2: min w

s.t.
$$2y_1 + 2y_2 \leq w$$
$$y_1 + 3y_2 \leq w$$
$$y_1 + y_2 = 1$$
$$y_1, y_2 \geq 0, \quad w: \text{free}$$
$$y_1 = 1, \ y_2 = 0, \ w = 2$$

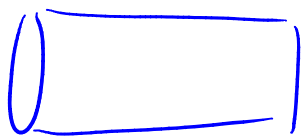
(alternative sol's exist)

Theorem (von Neumann '28)

Every $m \times n$ zero-sum game has a sol'n with v as the value of the game + and optimal (pure or mixed) strategies for P1 + P2

Ex. Cutting a cake (constant sum $\equiv$ zero-sum)

Ex. Cutting a cake (Constant Sum ≡ Zero-Sum)



Chooser

Cutter		Gets big	Gets small
	Even	$\frac{1}{2}, \frac{1}{2}$	$\frac{1}{2}, \frac{1}{2}$
	Uneven	s, l	l, s

$s + l = 1$

Ex. Tragedy of the Commons

Aussie farmers water use

Everyone else

Individual farmer		Cheat	Cooperate
	Cheat	2, 2	10, 5
	Cooperate (limited amt of water)	1, 2	5, 5

b) History of GT

c) Classification of games

- Non-coop.
  - Zero-sum
    - Diff. games
  - Non-zero sum
    - Diff. gams
    - Complete Info
      - Static
      - Dynamic
    - Incomplete Info
      - Static
      - Dyn.
      - Mech. design
        - Adverse sel.
        - Moral hazard
- Cooperat
  - Dominant set
  - Core
  - Shapley value
  - Nucleolus

d) Non-zero sum games

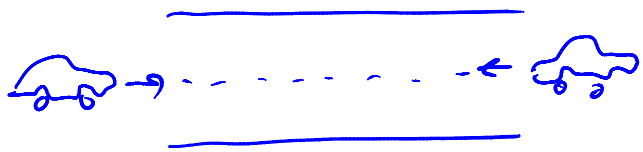
Ex. Battle of the sexes (BofS)

woman

Socc. Ball

$$\begin{array}{cc} & \\ \text{Man} \begin{array}{c} \text{Socc.} \\ \text{Ballet} \end{array} & \begin{pmatrix} 2,1 & 0,0 \\ 0,0 & 1,2 \end{pmatrix} \end{array}$$

Ex. Chicken



#1

$$\begin{array}{cc} & \begin{array}{cc} S & HT \end{array} \\ \begin{array}{c} \text{Swerve} \\ \text{Hang tuff} \end{array} & \begin{pmatrix} 1,1 & 0,2 \\ 2,0 & -1,-1 \end{pmatrix} \end{array} \qquad \begin{pmatrix} 1,1 & 0,2 \\ 2,0 & -100,-100 \end{pmatrix}$$

$$\text{mixed} \quad \begin{array}{cc} & \begin{array}{cc} 1/2 & 1/2 \end{array} \\ \begin{array}{c} 1/2 \\ 1/2 \end{array} & \begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix} \end{array} \qquad \begin{array}{cc} & \begin{array}{cc} .99 & .01 \end{array} \\ \begin{array}{c} .99 \\ .01 \end{array} & \begin{pmatrix} \quad & \quad \\ \quad & \quad \end{pmatrix} \end{array}$$

Ex. Prisoner's dilemma

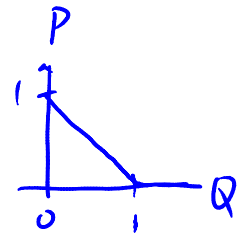
$$\begin{array}{cc} & \begin{array}{cc} \text{Not confess} & \text{Confess} \end{array} \\ \begin{array}{c} \text{Not confess} \\ \text{Confess} \end{array} & \begin{pmatrix} 1,1 & 9,0 \\ 0,9 & 6,6 \end{pmatrix} \end{array}$$

months in prison

Confess | 0,9 | 6,6 |

Ex. Cournot game

Two firms. Price $p = \max(1-Q, 0)$



Q: total quant

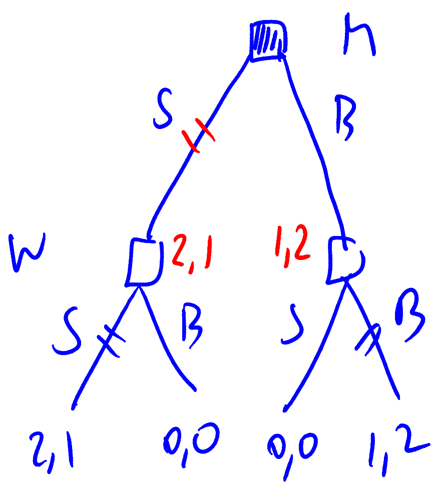
$q_1 + q_2 = Q$

$$K_i(q_1, q_2) = q_i(1-Q) = q_i(1-q_1-q_2)$$

Soln $q_1^* = q_2^* = \frac{1}{3}$

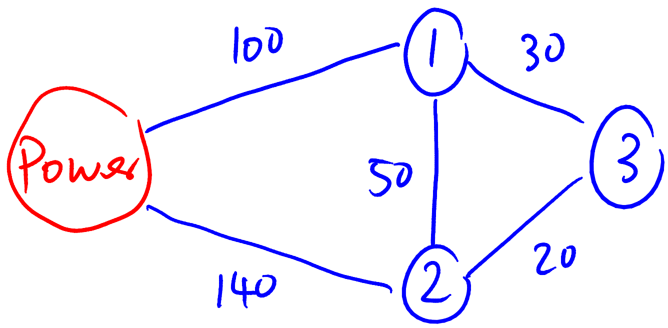
e) Extensive form game

Ex. Sequential B of S



f) Cooperative games

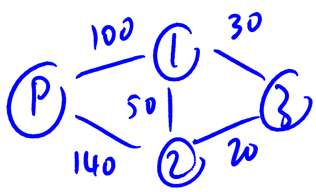
Ex. Three cities



$N = \{1,2,3\}$, coalition $S \subseteq N$

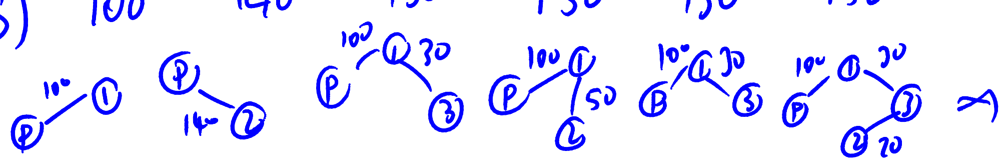
$c(S)$: cost of coalition S

$v(S)$: cost savings to " S



$$= \sum_{i \in S} c(\{i\}) - c(S)$$

S	$\{1\}$	$\{2\}$	$\{3\}$	$\{1,2\}$	$\{1,3\}$	$\{2,3\}$	$\{1,2,3\}$
$c(S)$	100	140	130	150	130	150	150
$v(S)$	0	0	0	$240-150$ $=90$	$230-130$ $=100$	$270-150$ $=120$	$370-150$ $=220$



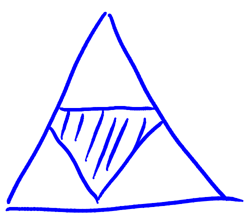
Sol'n for (N, v) : How to split savings?

$x = (x_1, x_2, x_3)$: imputations

$$x_1 + x_2 + x_3 = 220$$

One possibility. $x_1 = x_2 = x_3 = \frac{220}{3}$ No good!

Core.



Shapley value. $x = (65, 75, 80)$

Nucleolus. $x = (56\frac{2}{3}, 76\frac{2}{3}, 86\frac{2}{3})$

Start at 10:15 $\rightarrow$ 1:15