Example 2 (Ex2BM)

for: Reducing the Dimensionality of Linear Quadratic Control Problems

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Consider a somewhat ad hoc Keynesian macro model with a representative household/firm and nominal wage rigidities. All variables are in log terms. The government chooses the money supply and government expenditure to maximize the infinite horizon utility of the representative agent who likes consumption and dislikes work and inflation:

$$(23) \qquad U_t = -\sum_{\tau=t}^{\infty} (c_\tau - \overline{c})^2 + \lambda^2 l_\tau^2 + \kappa^2 (p_\tau - p_{\tau-1})^2 .$$

Given a binding cash-in-advance constraint:

$$(24) \qquad m_t = y_t + p_t ,$$

Aggregate demand for output is affected by government expenditure:

$$(25) \qquad y_t = f_0 + f_1 g_t .$$

In log terms, a Cobb-Douglas production function relates production to the inputs capital and labor:

$$(26) \qquad y_t = c_0 + c_1 k_t + c_2 l_t$$

Capital depreciates slowly and is affected by investment which is assumed to be exogenous but is stimulated by government expenditure (public infra structure):

$$(27) \qquad k_t = a_0 + \delta\, k_{t-1} + a_1\, g_t$$

Given nominal wages that are too high to clear the labor market and adjust slowly to market pressures (i.e., the price level and the amount of excess supply of labor) we have employment give as:

$$(28) \qquad l_t = d_0 + d_1 (p_t - p_{t-1}) - d_2 (l_{t-1} - \bar{l})$$

Lastly, consumption depends linearly on current income:

$$(29) \qquad c_t = b_0 + b_1\, y_t .$$

Choosing p_t, p_{t-1}, k_t, l_t and a constant as the state variables, some substitutions produce:

$$(30) \qquad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -d_1 & 0 & 1 \end{pmatrix} \begin{pmatrix} p_t \\ k_t \\ l_t \end{pmatrix} = \begin{pmatrix} f_0 & 0 & 0 & 0 & 0 \\ a_0 & 0 & \delta & 0 & 0 \\ d_0 & -d_1 & 0 & -d_2 & 0 \end{pmatrix} \begin{pmatrix} 1 \\ p_{t-1} \\ k_{t-1} \\ l_{t-1} \\ p_{t-2} \end{pmatrix} + \begin{pmatrix} 1 & -f_1 \\ 0 & a_1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} m_t \\ g_t \end{pmatrix} .$$

The “output” in terms of consumption, leisure, and inflation is given as:

$$(31)\qquad \begin{pmatrix} c_t - \bar{c} \\ -\lambda l_t \\ -(p_t - p_{t-1}) \end{pmatrix} = D \begin{pmatrix} 1 \\ p_t \\ k_t \\ l_t \\ p_{t-1} \end{pmatrix}, \text{ with } D = \begin{pmatrix} g_0 & 0 & g_1 & g_2 & 0 \\ 0 & 0 & 0 & -\lambda & 0 \\ 0 & -\kappa & 0 & 0 & \kappa \end{pmatrix}.$$

Thus, we obtain

$$(32)\qquad A = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ f_0 & 0 & 0 & 0 & 0 \\ a_0 & 0 & \delta & 0 & 0 \\ d_3 & -d_1 & 0 & -d_2 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}, \quad C = \begin{pmatrix} 0 & 0 \\ 1 & -f_1 \\ 0 & a_1 \\ -d_1 & d_1 f_1 \\ 0 & 0 \end{pmatrix}, \quad K = D'D$$

where $d_3 = d_1 f_0 + d_0$, $g_0 = -\bar{c} + b_0 + b_1 c_0$, $g_1 = b_1 c_1$, $g_2 = b_1$.

This problem can be reduced to scalar iteration. Run Ex2BM.m with RedMainFin.m for details.