# Determinants and Predictability of Commodity Producer Returns*

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**Abstract**

We derive stock returns for firms producing nonrenewable commodities employing the investment-based asset pricing approach. By identifying the appropriate time-varying discount rate the investment-based approach allows an alternative test of the Hotelling Valuation Principle. The empirical results support the principle and enable predicting returns from sorting firms into quintiles by expected return, producing a 16-20 percent realized difference between top and bottom quintile. The return differences cannot be explained by standard risk factors or a commodity-specific factor, suggesting that an important risk factor is still missing from standard models. The approach permits cost-of-capital estimation that circumvents identifying systematic risk factors.

***JEL Classification***: G12, G17, G11, C38.

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# Determinants and Predictability of Commodity Producer Returns

## 1. Introduction

We adapt the investment-based asset pricing approach of Cochrane (1991) to determine the stock returns for producers of non-renewable commodities. The approach allows prediction of returns and calculation of the costs of capital for these firms without the need to specify systematic risk factors.[1] The focus on non-renewable commodity producers facilitates the investment-based approach because the investment returns of these firms are driven by well-known principles of optimal exhaustible resource extraction. In-ground commodity reserves may be viewed as inputs to produce commodities, allowing direct calculation of investment returns. The investment-based approach then identifies how the firms' expected stock returns relate to investment returns and thus commodity reserves and other inputs.[2] [3]

The literature on optimal exhaustible resource extraction provides guidance for applying the investment-based approach. By the original Hotelling Rule (Hotelling, 1931) firms adjust resource extraction until the expected increase in the commodity spot price equals the risk-free rate. Building on previous modifications of the Hotelling analysis we determine the commodity-producing firm's output more generally where the marginal net profit from

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[1] Specifying the correct factors affecting average returns is problematic. Even the mainstay models of Hou, Xue, and Zhang (2015) and Fama and French (2015) struggle to explain the return variation within portfolios of test assets and leave numerous anomalies unexplained (Linnainmaa and Roberts, 2018, Jacobs and Müller, 2020).

[2] The investment-based approach has featured, apart from tangible capital, different inputs as determinants of stock returns. For instance, Lin (2012) finds that R&D investment predicts stock returns; Da, Huang, and Yun (2017) show that electricity usage predicts stock returns; and Belo et al. (2020) demonstrate that labor hiring predicts stock returns. Our approach adds commodity reserves to the input list.

[3] Two previous papers have related stock returns of commodity producers to production variables. Yang (2013) presents a production-based asset pricing model. Commodity producers are viewed as regular firms who manufacture non-storable consumption goods. Yang's approach does not treat commodity reserves as inputs and cannot treat commodities as nonrenewable. The approach also does not relate stock returns to investment returns and, accordingly, requires (exogenous) specification of a stochastic discount factor. Chen (2016) investigates the links between the stock returns of (an index of) commodity-influenced producers and commodity price increases. But examines the effect of exogenous stock returns on commodity prices, instead of the effect of exogenous commodity prices on stock returns. Chen does not consider additional determinants and works outside the production-based approach by taking the expected stock return as exogenous and constant.

extraction of the commodity equals the firm's required return. Identification of the required return on equity follows from the determinants of the firm's net profit margin.

Miller and Upton (1985) recast the Hotelling Rule into what they call the Hotelling Valuation Principle (HVP): given intertemporal profit maximization, the market equity of the firm as a fraction of its total reserves must equal the current commodity price net of the marginal extraction cost. Computing the present value of the firm, in principle, entails evaluating the future revenues from all reserves at the prices and profit margins prevalent at all future times when the reserves are sold. By the HVP, however, the only relevant price (and marginal extraction cost) to consider for valuing the entire reserve stock is today's price.

The HVP takes commodities prices as given and checks if firm valuation is consistent with the HVP. Subsequent research, however, concludes that it typically overestimates the value of proven and probable reserves by as much as fifty percent. Our approach looks at the HVP from a different angle. Instead of evaluating the valuation levels of the commodity-producing firms we examine the change in levels, captured by stock market returns. The difference is that omitted variables related to, for instance, market power, taxation, and real options may impact the level of firm value, but do not interfere with the intertemporal equalization of discounted profit margins if these variables are stable over time. From this perspective, the HVP will fail in level terms, but remain useful in difference terms, which we target by focusing on stock returns.

As a secondary motivation, consider that the commodity-producing firms represent a market segment that has interesting potential as a component in investment portfolios. The returns of these firms are, on the one hand, strongly tied to the underlying commodities.[4] On

[4] For instance, Tufano (1998), Baur (2014), Zhang (2015), and Dar, Bhanja, and Paul (2019).

the other hand, the underlying commodity prices are weakly or even negatively correlated with stock market returns in general.[5] The question is if the commodity-producing firms act more like the commodities or more like the general stock market, and how the connections change with aggregate fluctuations. These issues are of practical importance because they determine the effectiveness of these firms as part of diversified investment portfolios or their value as hedging instruments that may be more liquid than real commodities.

To preview the results, we find that the qualitative predictions of the modified Hotelling model directed to the explanation of stock returns are confirmed entirely for industrial metals and for energy resources, and partially for precious metals. The empirical results pertain to explaining the stock returns of the firms, but also to predicting their returns. We find that sorting the exhaustible commodity-producing firms into quintiles based on their expected stock returns, using only past information, and holding the top quintile while shorting the bottom quintile, yields an annualized return of around 16 percent for sorting based on prior pooled estimation and above 20 percent for sorting based on prior firm fixed effects estimation. These returns are hardly diminished by adjusting for standard risk factors, or for a commodity-specific risk factor. The fact that risk-adjusted returns are so large suggests that other, unidentified, systematic factors are central for commodity-producing firms. Unless the unidentified factors that are priced for the commodity-producing firms are non-systematic (i.e., only relevant for pricing commodities), unidentified systematic factors must be central for financial assets in general. [6]

[5] See Pindyck and Rotemberg (1990), Zapata, Detre, and Hanabuchi (2012), and Daskalaki, Kostakis, and Skiadopoulos (2014). Commodity prices also forecast general stock returns. See Huang and Kilic (2019).

[6] Recent literature has identified factors which explain co-movement in commodities prices. Bakshi, Gao, and Rossi (2019), Boons and Porras Prados (2019), and Szymanowska (2014) find common factors related to momentum, basis, and basis-momentum. These, however, do not explain average stock returns.

## 2. Theoretical Development

### *2.1. Background*

Our theoretical development builds on two separate literatures: the Hotelling approach for optimal exhaustible resource extraction and investment-based asset pricing. The latter allows the appropriate discount rate (cost of capital) to replace the risk-free rate usually employed in the Hotelling framework.

By the original Hotelling Rule (Hotelling, 1931) firms adjust resource extraction until the expected increase in the commodity spot price equals the risk-free rate. This implies that commodity prices should increase monotonically over time. Various modifications of the Hotelling analysis, however, determine the commodity-producing firm's output more generally with different implications.

Whereas Hotelling did not model extraction costs, Miller and Upton (1985) discuss the extraction cost as depending positively on current output and negatively on the level of reserves.[7] The motivation of the latter is the Ricardian principle of mining the cheaper resources first. Hence, as reserves decrease, the marginal extraction costs increase. In deriving the HVP they set the firm's discount rate equal to the real interest rate, tacitly assuming the firm does not face systematic risk.

Slade and Thille (1997) extend the theoretical contribution of Gaudet and Khadr (1991) to move beyond viewing the discount rate as risk free and employ an arbitrage argument to establish the appropriate discount rate as the CAPM-based required return. Although this

[7] Slade (1982) had previously formally incorporated the dependence of costs on the reserve level, as well as modeling technological progress in extraction. This modified the Hotelling Rule, allowing commodities prices in equilibrium to follow a U-shaped pattern increase over time.

contribution allows firm-specific discount rates that account for systematic risk, the discount rates are determined exogenously and are constant over time.

Further refinements of the Hoteling approach include technological progress as considered by Lin and Wagner (2007), building on Slade (1982). Ellis and Halvorson (2002) incorporate the impact of market power in commodities markets. Slade (1984) considers government regulation and taxation affecting the value and decisions of mining firms. Shumlich and Wilson (2009) argue that the reserve values are lower than predicted by the HVP due to the existence of real options. Cairns and Quyen (1998) tackle the additional investment decisions of mining firms for exploring additional reserves.

Anderson, Kellogg, and Salant (2018) for the oil industry distinguish production from current wells and investment in drilling additional wells. They view the maximum output from a well as constrained by a fraction of total reserves in the well. Thus, increased output may arise from the intensive margin (more production from current wells, unless the constraint binds) and the extensive margin (drilling more wells). The Hotelling Rule is then amended to entail that the marginal return to drilling must rise at the discount rate.

We present an equilibrium model for exhaustible resource companies (mining firms) to derive the firm-specific elements determining their stock returns. The model contributes to the Hotelling (1931) setting by allowing for endogenous time-varying discount rates using the production-based asset pricing approach.[8] Utilizing the investment-based version of Cochrane (1991), Restoy and Rockinger (1994), and Liu, Whited, and Zhang (2009), we derive how stock

[8] Established by Cox, Ingersoll, and Ross (1981), Brock (1982), Berk, Green, and Naik (1986), Balvers, Cosimano, and McDonald (1990), Cochrane (1991, 1996), and Zhang (1995).

returns for the commodity-producing companies are related to their investment returns determined in the Hotelling framework.

The mechanism by which investment returns become related to stock returns is internal arbitrage by managers at the individual firm level. The firm takes both commodity prices and financial market prices as given. The expected stock return (cost of equity capital) viewed by management is revealed in the expected investment return which we can predict in advance for the individual (mining) companies. The link between stock returns and investment returns holds irrespective of what the risk factors are or how firm value loads on the risk factors. Accordingly, the resulting expected returns may be determined without knowing what the systematic risk factors are. Moreover, the investment and production decisions of firms that we focus on are guided by relatively transparent profitability considerations in place of the investments decisions of households guided by the more opaque utility considerations of the consumption-based asset pricing approach (See e.g., Lin and Zhang, 2013, and Zhang, 2017).

*2.2. The Model*

Consider a firm producing a nonrenewable commodity. The firm is competitive and takes the market price at time $t$ of its resource (commodity) $q_t$ as given. As in the Hotelling model, we assume the reserve level is finite and known, so the extraction ("production") quantity at time t, $y_t$, is equal to the difference in the reserve level $x_t$ between two consecutive time periods, i.e., $y_t = x_t - x_{t+1}$.

The cost-of-extraction function, $c(y_t, x_t)$ is assumed to be homogeneous of degree one in the production level and the reserve level, strictly convex increasing in production, $c_y(y_t, x_t) > 0$, $c_{yy}(y_t, x_t) > 0$ (single and double non-time subscripts indicate first and second partial derivatives, respectively), and decreasing in the reserve level, $c_x(y_t, x_t) < 0$. Extraction is costly, and marginal costs are increasing in the extraction amount. Extraction also is relatively easier and cheaper when there is a larger quantity of reserves. If both production and reserves increase by a particular percentage then the mining costs increase by the same percentage, which is implied by the homogeneity assumption.

The assumption that production costs be homogeneous of degree one is not commonly made in the literature on optimal resource extraction. However, it is necessary to apply the investment-based asset pricing approach. The assumption superimposes the reasonable requirement that the average production costs increase monotonically in the ratio of production to reserves, $y_t / x_t$. However, it rules out "well effects" as we discuss later. For later use we state here that, by the Euler homogeneous function theorem (see e.g., Varian 1992, pp. 481-482), $c(y_t, x_t) = y\, c_y(y_t, x_t) + x\, c_x(y_t, x_t)$, and that the first partial derivatives are homogeneous of degree zero: $c_y(y_t, x_t) = c_y(y_t / x_t, 1) > 0$ with, furthermore, $d\, c_y(y_t / x_t, 1) / d(y_t / x_t) > 0$, and $c_x(y_t, x_t) = c_x(y_t / x_t, 1) < 0$, while no restriction is imposed on the sign of $d\, c_x(y_t / x_t, 1) / d(y_t / x_t)$.

The commodity-producing company is assumed to issue riskless debt, $b_t$, which is renewed each period. All operating profits, net of the interest on the debt and the revenue from additional debt issuance, are disbursed to the shareholders as dividends, which accordingly equal:

$$d_t = q_t\, y_t - c(y_t, x_t) - r_{t-1}\, b_t + (b_{t+1} - b_t)\,, \tag{1}$$

where $r_{t-1}$ is the risk-free rate at time $t$ (pre-determined at time $t-1$). The cum-dividend market value of the mining firm to its shareholders is given as

$$V(s_t) = E_t\left(\sum_{j=0}^{\infty} m_{t+j}\, d_{t+j}\right), \tag{2}$$

where (with some abuse of notation) the cumulative stochastic discount factor between time *t* and time *t+j* is given by $m_{t+j}$ which is determined at the aggregate level and, for expositional simplicity is considered exogenous here as in Berk et al. (1986), even though its value and determination do not affect the ultimate solution for expected returns; $s_t$ indicates a set of state variables at time *t*. The commodity-producing firms are price takers in financial as well as commodity markets.

The firm's optimal extraction decision problem is expressed by the Bellman equation:

$$V(s_t) = \underset{y_t,\, b_{t+1}}{Max} \left\{ q_t y_t - c(y_t, x_t) + b_{t+1} - (1+r_{t-1})b_t + E_t[m_{t+1}V(s_{t+1})] \right\}. \tag{3}$$

Here $s_t = \{x_t, b_t, M_t\}$, with the first two firm-specific state variables and $M_t$ reflecting any number of macro state variables (including $q_t$ and parameters affecting the distribution of the stochastic discount factor and future commodity prices). The firm-specific equations of motion are

$$x_{t+1} = x_t - y_t \ , \qquad q_{t+1} = h(q_t, \varepsilon_{t+1}). \tag{4}$$

The second equation indicates that the commodity prices follow a stochastic process exogenous to the firm and with $\varepsilon_{t+1}$ a random variable with distribution parameters included in $M_t$. The implicit function theorem implies that the state variables $s_t$ in the Bellman equation are the non-choice variables (pre-determined or exogenous) in the decision problem of equations (3) and (4): the reserve level $x_t$, firm debt $b_t$, and the commodity price $q_t$. Any additional parameters determining the distribution of the stochastic discount factor $m_{t+1}$ and future commodity prices $\varepsilon_{t+1}$ also impact the state (together with $q_t$ captured by $M_t$).

Along the lines of Restoy and Rockinger (1994) the Appendix uses the first-order conditions, properties of the homogeneous cost function, and other constraints of the model in equations (1) thru (4) to obtain

$$E_t\left[m_{t+1}\left(\frac{q_{t+1}-c_y(y_{t+1},x_{t+1})-c_x(y_{t+1},x_{t+1})}{q_t-c_y(y_t,x_t)}\right)\right]=1\,. \tag{5}$$

The term in parentheses may be interpreted (following Cochrane, 1991) as the gross investment returns of the firm – the marginal return to leaving an extra unit of the commodity in the ground instead of mining it:

$$1+r_{t+1}^I=\frac{q_{t+1}-c_y(y_{t+1},x_{t+1})-c_x(y_{t+1},x_{t+1})}{q_t-c_y(y_t,x_t)}\,. \tag{6}$$

The interpretation is that the denominator represents the marginal cost of investing (leaving an extra unit in the ground), equal to the opportunity cost of forgoing the margin, $q_t-c_y(t)$. (Note that here and subsequently we use the single function argument "*t*" as short-hand notation for all function argument values at time *t*). The numerator represents the marginal benefit (discounted by $m_{t+1}$) of investing (extracting in the next period), $q_{t+1}-c_y(t+1)-c_x(t+1)$: the revenue in the next period net of the next-period marginal extraction cost, $q_{t+1}-c_y(t+1)$, which is mitigated, compared to what it otherwise would have been in equilibrium, by $-c_x(t+1) > 0$ because keeping more available reserves lowers the cost of extraction.

Equation (6) represents a modified Hotelling Rule: the commodity spot price $q_t$, when adjusted for marginal production costs $c_y(t)$ and the marginal cost impact of resource depletion $c_x(t)$, grows at the investment hurdle rate $r_t^I$. This would occur in general equilibrium if all firms were similar. The implication of the Hotelling Rule then is that spot prices rise over time (especially in Hotelling's original formulation when marginal production costs are ignored) which is easily refuted by the observation that spot prices of most commodities have not monotonically increased over time.[9] Our focus is on the other direction, in which we take as given a stochastic path for commodity prices and use that to explain stock returns.

Miller and Upton (1985) introduced the idea of applying the Hotelling Rule in reverse by using the optimal intertemporal production decisions to value a commodity-producing firm by what they call the Hotelling Valuation Principle (HVP). For our model, the Appendix derives a solution for the (ex-dividend) value of the firm:

$$p_t + b_{t+1} = [q_t - c_y(y_t / x_t)]\ x_{t+1}\,. \tag{7}$$

The value of the firm, the market value of equity $p_t$ (the number of outstanding shares is normalized to one) and debt $b_{t+1}$ together, in equilibrium equal the total end-of-period reserves $x_{t+1}$ times the current marginal unit profit margin $q_t - c_y(y_t / x_t)$, which is the commodity spot price net of the marginal cost of producing (extracting) the commodity. Miller and Upton

[9] See for instance Livernois (2009) and Schwerhoff and Stuermer (2019). The empirical evidence further is mixed about the performance of the various augmented versions of the Hotelling Rule (Livernois, 2009, Slade and Thille, 2009).

(1985) use equation (7) (though with constant marginal costs) to test the HVP. They neatly confirm the HVP by finding empirically that a linear regression of $(p_t + b_{t+1}) / x_{t+1}$ on $q_t$ generates a slope close to *one*.

Subsequent research, however, shows less support for the HVP. Adelman (1993) concludes that the predicted reserves are only about half of the measured reserves: linear regression of $(p_t + b_{t+1}) / x_{t+1}$ on $q_t$ generates a slope of barely above one-half.[10] The reason may be limitations of the model (discussed at the end of this section) but may also relate to the accounting method for reserves – how it incorporates probable and possible reserves and the potential for developing and discovering reserves, and whether it has a conservative bias. By focusing on *differences* in the market value across firms or over time, as measured by stock returns, we move away from assessing the *level* of the market value of the firm. If accounting biases are stable, focus on returns instead of prices will avoid the bias.

Equation (7) holds based on a cost function like that used in Slade and Thille (1997), but with an additional restriction of linear homogeneity imposed to be able to apply the investment-based asset pricing approach. Accordingly and notably, equation (7) is derived taking into consideration the relevant cost of capital of the firm. The contributions of Gaudet and Khadr (1991) and Slade and Thille (1997) allow for the important addition of risk and risk-based discounting in the Hotelling framework but they take the firm's discount rate as constant over time. Slade and Thille apply the CAPM to obtain empirical estimates of the firm's discount rate and then confirm that the spot price dynamics of the commodity are consistent with the Hotelling Rule. Our intent is the reverse. Considering the spot price dynamics of the

[10] See also Adelman (1990), McDonald (1994), Cairns and Davis (2001), and Shumlich and Wilson (2009).

commodity and firm production decisions we explain and predict the discount rate, i.e., the stock return.

The previous literature has not considered endogenous firm-level returns in this context. Employing the definition of the gross market return on the firm's equity, $1+r_{t+1}^{S}=(p_{t+1}+d_{t+1})/p_t$, we can derive directly from equations (1) and (7), and given that $c(y_t,x_t)=c_y(y_t/x_t)\,y_t+c_x(y_t/x_t)\,x_t$ for a homogeneous cost function:

$$r_{t+1}^{S}-r_t=\frac{[q_{t+1}-(1+r_t)\,q_t]+[(1+r_t)\,c_y(y_t/x_t)-c_y(y_{t+1}/x_{t+1})-c_x(y_{t+1}/x_{t+1})]}{q_t-c_y(y_t/x_t)-(b_{t+1}/x_{t+1})}. \tag{8}$$

The excess return expression is best understood with reference to the investment return. Given the assumption that the cost function is homogeneous of degree one, it is known from Hayashi (1982) that average and marginal returns are equal. Therefore, the (marginal) investment return is equal to the overall (average) return on assets.[11]

By optimization (internal arbitrage) the firm will continue to invest until the investment return equals the cost of capital. It follows that observing the investment return is equivalent to observing the cost of capital as management perceives it to be. Thus, the factors that affect the investment return also affect our assessment of the asset return. In equilibrium, the factors determining the investment return exactly identify the cost of capital. As first pointed out by

[11] The excess stock return expression in equation (8) applies to firms processing exhaustible inputs and may be compared to the excess returns for the "normal" firms covered by Cochrane (1991, equation 15). These depend on investment-to-capital ratios $I_{t+1}/K_{t+1}$, $I_t/K_t$, and a stochastic marginal product of capital. To determine the dynamics of the capital stock and satisfy second-order conditions, the derivation requires capital adjustment costs that are convex and homogeneous of degree one in $I_t$ and $K_t$. In our exhaustible-resource case, identifying the investment return does not require specifying the endogenous process of capital accumulation because the input is already in place and is reduced by the quantity of the commodity processed. It would be useful to benchmark the explanatory power of our exhaustible commodities model to that for normal firms. However, the two formulations have no determinants in common and in previous empirical work the investment-based approach has not examined returns of individual firms but rather has focused on portfolio returns.

Cochrane (1991, 1996), the investment returns are equal to the stock returns. This is not exactly the case here because the firm need not be fully equity financed. We have investment return equal to return on assets, $r_t^I = r_t^A$, and the excess return on assets is equal to the equity fraction of firm value times the excess stock return, as in Liu et al. (2009): $r_t^A - r_{t-1} = (1-\lambda_{t-1})(r_t^S - r_{t-1})$, where $\lambda_{t-1} \equiv b_{t+1} / (p_t + b_{t+1})$.[12] This follows directly from equations (6), (7), and (8). Hence, the excess stock return in equation (8) is simply the levered excess investment return. The assumption of a homogeneous cost function allows us, empirically, to replace the difficult-to-observe investment hurdle rate by a linear function of the observable stock return.[13]

The intuition for excess stock returns equation (8) is as follows. The first term in brackets (the basic Hotelling's Rule term) indicates that the impact on excess return is positive whenever $(q_{t+1} - q_t) / q_t > r_t$. All else equal, when commodity prices grow faster than the risk-free rate, then investment returns increase since the benefit of leaving the commodity buried until the next period rises. The second term in brackets indicates that strictly increasing marginal cost of extraction, $c_y(t+1) > c_y(t)$, negatively affects stock returns. It does, since, along the optimal production path, the future higher marginal production costs imply a lower investment return. In addition, the inexorable reduction over time of reserves implies a lower investment return as it raises the production costs, $-c_x(t+1) > 0$. The final term, the denominator, indicates the current benefit of producing instead of investing. If it is lower, the investment return is higher. Alternatively, this denominator term also equals, from equation

[12] Harris and Pringle (1985) and Cooper and Nyborg (2006) show that, if the firm continuously adjusts its capital structure to a fixed leverage ratio, this equation is the correct way to relate stock returns and equity returns, even if taxes are involved.

[13] This is analogous to a similar assumption that allows unobservable marginal Q to be replaced by observable average Q in Tobin's investment analysis (Tobin, 1969)

(7), the (scaled) equity value, which is associated with more leverage and so higher returns if the term is lower.

The explanation for the stock returns associated with the production-based asset pricing approach differs from that of the consumption-based approach which requires discussion of risk premia and exposure to the risk factors. In principle the results should be consistent. The stock return differences across firms should be related to differences in loadings on the risk factors that we did not need to specify in our production-based approach. Higher stock returns in equation (8), because of lower anticipated production growth (lower marginal costs) or higher commodity price growth (increasing revenue), means higher investment returns so that current production is lower and more of the commodity is left in the ground. From the alternative consumption-based perspective the increased reserves and uncertainty about their future value imply the firm is more exposed to just about any systematic risk, requiring higher stock returns. In view of the difficulty in the literature to identify common systematic risk factors, a major advantage of our approach is that we do not need to identify these factors.

Our model specification has some limitations. The model ignores taxes and regulation (see Slade, 1984), monopoly power in commodity markets (see Ellis and Halvorson, 2002), and real options (see Shumlich and Wilson, 2009). More significantly investment in exploration is missing from the model (see Cairns and Quyen, 1998). The model also does not allow for re-evaluation of economic accessibility of reserves when commodity prices or mining technology change. As a result, reserve quantities may only decrease. However, 56% of the firms in our sample report at least one increase in the quantity of reserves over the sample period. To address this issue, we may extend the model based on Pindyck (1978), which considers exploration, to include the time-varying discount rate. It would generate the same determinants of equity returns as our featured model plus an extra factor which is the expected marginal benefit of exploration per unit of exploration cost. The excess return expression in equation (8) omits this variable.

A further limitation of the model is related to the assumption of linearly homogeneous costs which we impose to apply the investment-based asset pricing approach. The assumption implies that unit production costs are a function of the ratio of production to reserves only and hence cannot easily account for issues such as changes in well pressure: In practice, especially for crude oil extraction, independently of the level of reserves, a high extraction speed raises the unit production costs disproportionately (or is simply impossible) at the well level as emphasized by Anderson, Kellogg, and Salant (2018). This would generate lower investment returns and cause marginal returns to deviate from average returns, which our approach does not permit.

Model limitations may be responsible for firm-level market valuation not conforming to the HVP in empirical tests. Our theoretical derivation of equilibrium firm-level stock returns in the Hotelling framework allows the different perspective of focusing on changes in market valuation. Accordingly, relatively stable deviations from valuation due to the model limitations will cancel; or if value biases are not stable but unpredictable, they show up as random return shocks. On the other hand, commodity-producing firms have in common the tradeoff between current and future liquidation of their assets as an essential driver of profitability. We believe that the associated incentives are reasonably well captured by our modified Hotelling framework and constitute key determinants of stock returns.

## 3. Implications and Empirical Specification

The results from the model may be summarized in the following proposition:

**PROPOSITION.** Dynamic maximization of profitability by a commodity-producing firm of equation (2) subject to equations (1) and (4), and given a cost function $c(y, x)$ that is homogeneous of degree 1 in reserves $x$ and output $y$, implies that the firm's expected excess stock returns are given by:

$$E_t(r_{t+1}^S) - r_t = \frac{(q_{t+1}/q_t) - (1+r_t) + \{(1+r_t)c_y(y_t/x_t) - E_t[c_y(y_{t+1}/x_{t+1}) + c_x(y_{t+1}/x_{t+1})]\}/q_t}{p_t/q_t\, x_{t+1}} \quad (9)$$

Here subscripts represent either time $t$ or a derivative with respect to the indicated function argument. Further, $r$ is the risk-free return; $r^S$ is the stock return and $p$ the market value of the firm's stock; $q$ is the commodity price.

Given auxiliary assumptions:

$$c(y_t, x_t) = (c/2)\, y_t^2 / x_t \ , \quad Var_t(y_{t+1}/x_{t+1}) = V \text{ for all } t\,, \quad (10)$$

six variables affect returns, with directions as follows:

| VARIABLES | $E_t q_{t+1}$ | $q_t$ | $r_t$ | $\frac{E_t y_{t+1}}{x_{t+1}}$ | $\frac{y_t}{x_t}$ | $\frac{q_t\, x_{t+1}}{p_t}$ |
|---|---|---|---|---|---|---|
| $E_t(r_{t+1}^S - r_t)$ | + | − | − | − | + | + |

(11)

*Proof.* Taking expectations in equation (8), using equation (7) to replace the denominator by $p_t / x_{t+1}$, and then dividing numerator and denominator by $q_t$, yields equation (9). We may work with $E_t y_{t+1}/x_{t+1}$ (instead of $E_t[c_y(y_{t+1}/x_{t+1})]$) if the marginal cost function is assumed to be linear (i.e., costs are quadratic). So, for empirical purposes it is convenient to assume $c(y_t, x_t) = (c/2)\, y_t^2 / x_t$ as in equation (10). This specific cost function satisfies the conditions we discussed, including homogeneity. Applying equation (10) to (9) gives

$$E_t r_{t+1}^S - r_t = \left( \frac{E_t q_{t+1} - q_t}{q_t} - r_t - \frac{c}{q_t} \left[ \frac{E_t y_{t+1}}{x_{t+1}} - \frac{1}{2} (\frac{E_t y_{t+1}}{x_{t+1}})^2 - \frac{1}{2} Var_t (\frac{y_{t+1}}{x_{t+1}}) - (1 + r_t) \frac{y_t}{x_t} \right] \right) \frac{q_t \, x_{t+1}}{p_t} \qquad (12)$$

Apart from the production variance, which we assume constant, $Var_t(y_{t+1} / x_{t+1}) = V$, in equation (10) six variables affect returns.[14] The impact directions are indicated in (11). □

The first two reflect opposite directions of the revenue effects from producing now compared to the next period. The second two relate to opposite directions regarding the marginal costs of producing now compared to the next period. The fifth variable represents the Hotelling value of reserves relative to the market value of equity. Since $q_t \, x_{t+1} / p_t = [1/(1-\lambda_t)]\{1/[1-(c_y(t)/q_t)]\}$ from equation (7), a higher value indicates a combination of higher financial leverage $\lambda_t$ (raising stock returns for given investment returns) and lower current profit margin $c_y(t)/q_t$ (raising investment returns) both implying higher stock returns. From equation (8), the impact of the final determinant, the risk-free rate, is to raise the opportunity cost of investing. The impact on the excess stock return is proportionate to $-[q_t - c_y(y_t / x_t)]$ which must be negative. Only the sign of the link between $\frac{E_t y_{t+1}}{x_{t+1}}$ on $E_t(r_{t+1}^S) - r_t$ is not mathematically obvious. Given the cost function of equation (10) the negative sign requires $\frac{E_t y_{t+1}}{x_{t+1}} < 1$ which holds as production cannot exceed total reserves.

To predict returns, i.e., generate $E_t(r_{t+1}^S)$, by equation (12) both $\frac{q_{t+1}}{q_t}$ and $\frac{y_{t+1}}{x_{t+1}}$ need to be forecast. First, growth in commodity prices is predictable but the anticipated growth rate may change over time. We obtain the forecast as the mean growth rate based on the 36 previous

[14] Since we can only work with annual data on the production and reserve variables it is not practical to allow for time variation in the conditional production variance.

monthly observations. Although we do not derive this formulation from first principles, it allows the forecast of the growth rate to vary over time:[15]

$$E_t\, q_{t+1} / q_t = (1/S) \sum_{s=1}^{S} q_{t+1-s} / q_{t-s} \,, \qquad S = 36\,. \tag{13}$$

Second, to forecast $y_{t+1} / x_{t+1}$ we run a linear regression, using only data up to time $t$ to forecast $y_{t+1} / x_{t+1}$ for time $t+1$. Our production forecast $E_t(y_{t+1} / x_{t+1})$ is then used to forecast the stock return $r_{t+1}^{S}$, using exclusively past information. The forecast variables in this regression are the same as those used in equations (9) and (12):

$$E_t\, y_{t+1} / x_{t+1} = f\,[E_t\, q_{t+1},\; q_t,\; r_t,\; y_t / x_t,\; q_t x_{t+1} / p_t]\,. \tag{14}$$

In the following we estimate equation (12) using equations (13) and (14).

## 4. Data

### *4.1. Sample selection*

We use all mining firms in the *Compustat Industry Specific Annual* database with available production and reserve data. Compustat contains operational data for North American companies in particular industries including airlines, gaming, mining, oil and gas, etc. The

[15] In an earlier draft we approximated the expected future spot rate by the futures rate, assuming a constant bias in the futures rate as a forecast of the future spot rate, but there are problems with this proxy. First, the futures rate and bias are endogenous and both stock returns and the futures rate may be affected by extraneous variables. E.g., investment may lower both stock returns and the commodity futures rate (as in David 2019, e.g., for the oil market). Second, the risk premia giving rise to the bias are not constant, as shown, for instance, by Szymanowska et al. (2014). Akin to this, as emphasized in Koijen et al. (2018), the futures premium (our earlier proxy) is closely related to the carry (or basis), whereas the expected commodity price appreciation (the variable we are seeking to approximate) is the complementary component of the expected commodity return that may not be highly correlated with the carry. Our empirical results when the futures rate proxies for the expected future spot rate are in *Web Appendix W1*. Compared to our main results they are very similar except that the coefficient on the expected commodity price increase is quantitatively smaller. Just as for risk premia of stocks, there is considerable discussion about which systematic risk factors explain commodity price risk premia. See Bakshi et al. (2019), Beckmann, Belke, and Czudaj (2014), Boons and Porras Prado (2019), Daskalaki et al. (2014), Ratti and Vespignani (2015), Szymanowska et al. (2014), and Yang (2013, p.165).

mining industries covered include gold, diversified metals and mining, precious metals and minerals, and oil and gas. The products include gold, silver, copper, nickel, zinc, coal, metallurgical coal, iron ore, oil, natural gas and natural gas liquid. These are all exhaustible resources subject to the theoretical forces of the Hotelling analysis.

We exclude firms whose main mining products are iron ore, coal and met coal since they are not highly liquid commodities in North America.[16] Our sample consists of the North American firms for which COMPUSTAT has annual data on reserves for the fiscal year ending in calendar year $t-1$ and have production data on one or more of the following commodities: gold and silver (precious metals), copper, nickel and zinc (industrial metals), WTI crude oil, and natural gas (energy fuels) for calendar year $t$. Because the industry-specific data from Compustat are available from 1999 forward, our return sample starts in July 2001 (since we use data from the previous full calendar year 2000 for reserves and production to explain returns in 2001) and ends in December 2018, containing a total of 52,337 firm months that meet our data criteria. Reserves for each firm are measured as the sum of their *proven* reserves and *probable* reserves (excluding *possible* reserves).[17]

Stock returns and market equity at the monthly frequency are also from Compustat for both the commodity-producing firms listed on Canadian and United States exchanges. They

[16] The lack of liquidity means that quoted commodity prices may not reflect fundamentals well at each time. These same commodities are also excluded in other recent asset pricing studies involving commodities. E.g., Bakshi, Gao, and Rossi (2019), Bianchi, Drew, and Fan (2016), Boons and Porras Prado (2019), Daskalaki et al. (2014), Gorton, Hayashi, and Rouwenhorst (2013), Koijen et al. (2018), and Szymanowska et al. (2014).

[17] For the mineral mining commodities (gold, silver, copper, zinc, and nickel in our case) Compustat provides "proven and probable reserves" whereas for oil and gas it provides "total proved reserves". Both are similar: SEC reporting guidelines define proven and probable reserves as deposits that may be economically and legally extracted at the time of reserve determination, and total proved reserves as recoverable with reasonable certainty under existing economic and operating conditions (Securities and Exchange Commission, 2021, Subparts 229.1200 and 229.1300). The criterion of economic viability implies that recorded reserve quantities may vary based on commodity prices, a possibility that our model ignores. For 58% of the firms in our sample, their reserves increase at some point during our sample period. The reason may be increased economic viability, or it may be the result of exploration.

are computed from end-of-month closing prices adjusted for dividends and stock splits. For dual-listed firms we use the listing in the country of origin. Monthly real risk-free returns are measured by the US 3-month T-Bill rate available from the Board of Governors of the Federal Reserve System minus the realized CPI-based (urban consumers, seasonally adjusted) inflation rate from the Bureau of Labor Statistics.

Table 1 provides an overview of the available data. Panel A lists the number of firms producing each of the commodities by country (U.S., Canada, and Other, where "Other" consists of any mining firms listed on North American exchanges but incorporated outside of the U.S. and Canada). In terms of the main activity of the companies, the sample is dominated by fuel energy firms, 693 in total, while there are 110 precious-metal producing firms and 28 industrial-metal producers (on average over time). Panel B shows the total number of included firms by year (2001-2018) and by country (US, Canada, and Other). The minimum number of included firms is 176 in 2001 and the maximum is 328 firms in 2014.

### *4.2. Predictor variable construction*

To obtain the relevant commodity prices at the firm level, we collect monthly commodity data from *Bloomberg* for the seven commodities in our sample. These commodity data include spot prices for gold, silver, copper, nickel and zinc, and nearest-to-maturity futures data for the energy fuels, oil and gas.[18] As most of the firms in our sample produce multiple commodities, a firm-level commodity price is calculated as the production-weighted average value of the individual commodity prices. The weights for each commodity are calculated with

[18] Because spot prices are not available from exchanges for the energy fuels it is common to use nearest-to-maturity futures data instead. See, for instance, Litzenberger and Rabinowitz (1995) and Khan, Khokher, and Simin (2017). To check that this convention does not distort our results relative to other commodities we also obtain the regression and sorting results when all commodity prices are replaced by nearest-to-maturity futures prices. These results deviate little from our main results and are presented in *Web Appendix W3*.

production quantity and sales price data reported in year $t-1$ since the production data are available only on an annual basis.

Similarly, the variables $q_t x_{t+1} / p_t$ (current value of reserves as a fraction of market equity) and $y_t / x_t$ (production as a fraction of total reserves) are obtained as production-weighted average values since the reserve estimation data and production information are available for each company at the product level instead of at the consolidated company level. We use each firm's market equity at the end of December of year $t-1$ to compute $q_t x_{t+1} / p_t$. Note that $x_{t+1}$ is determined at time $t$ once the current production level is deducted from reserves. Although we abstract from exploration and revaluation of reserve quantities in the model, reappraisals of the value of reserves are included in our empirical measure.

*4.3. Summary statistics*

Table 2 provides descriptive statistics of the returns and predictor variables. The effective sample period is from July 2001 to December 2018. Panel A presents the summary statistics for the full sample, aggregated across the seven commodities. The average returns over the sample period are 0.59 percent a month with a standard deviation of 15.21 percent. The average monthly interest rate is 0.11 percent in this period. The average one-month-ahead commodity spot price growth rate is 0.87 percent with a standard deviation of 1.40 percent. The average production-to-reserves ratio over the sample period is 0.127 (i.e., current production is on average about 13 percent of the proven and probable reserves) with a standard deviation of 0.076. The average value-of-reserves-to-market-equity ratio is 4.65 (which is the product of a leverage term and a reserves-to-firm-value term) with a standard deviation of 5.33. As an indicator of leverage, the average value of debt in ratio to the value of the firm is 0.34.

Panels B-D of Table 2 for the same variables provide the statistics separated into precious metals, fuel energy, and industrial metals.[19]

## 5. Empirical Results

To estimate equation (12) for the forecasts of the stock returns of the mining firms we first obtain the independent variables. At time $t$ the variables are the commodity spot price $q_t$, the quantity of production (as a fraction of prior reserves), $y_t / x_t$, the value of reserves per unit of stock market value, $q_t x_{t+1} / p_t$ (note that $x_{t+1} = x_t - y_t$ is known at time $t$), and our measure for the risk-free rate, $r_t$. In addition, known at time $t$, are the commodities price forecast, $\widehat{q_{t+1}} \equiv E_t(q_{t+1})$, and the production level forecast, $\widehat{y_{t+1} / x_{t+1}} \equiv E_t(y_{t+1} / x_{t+1})$.

### *5.1. The production-to-reserves forecast*

Forecast the production level as a fraction of reserves by linearizing equation (14):

$$\widehat{\tfrac{y_{t+1}}{x_{t+1}}} = \widehat{a_{0t}} + \widehat{a_{1t}} \ln(\widehat{q_{t+1}}) + \widehat{a_{2t}} \ln(q_t) + \widehat{a_{3t}}(r_t) + \widehat{a_{4t}}(\tfrac{y_t}{x_t}) + \widehat{a_{5t}}(\tfrac{q_t x_{t+1}}{p_t}) \tag{15}$$

Notice that the coefficients change for each period because only past information is used to obtain predicted values at each time. The estimates of equation (15) by OLS at the annual frequency are shown for the full sample only in Table 3, regression (4). The estimation results are presented for both a pooled specification, in which all mining firms are treated equally irrespective of commodity produced or home country (US, Canada, or "Other"), and a panel specification allowing individual firm effects. Separation by commodity is hard because most firms in our sample produce multiple commodities jointly.

[19] The classification is like that of the Institute for Financial Markets. The three categories we use are the exhaustible resource categories of the seven used by Szymanowska et al. (2014). (Compared to Szymanowska et al. we drop the food-based categories – Meats, Grains, Oilseeds, and Softs).

In both specifications lagged production is significant at the 1% level with a coefficient of 0.91 indicating a high level of persistence in production in the pooled case, and 0.46 in the panel case. The reserves-to-equity variable, acting essentially like a Tobin's Q variable in this context, is also significant at the 1% level, with a coefficient of -0.0016 in the pooling case and -0.0021 in the panel case. It reflects higher production in anticipation of profitable opportunities.

*5.2. Stock return prediction regressions*

Using only past information, stock returns for the following month can be predicted from equation (9). The production and reserve data are annual whereas the other variables – stock returns, commodity prices, and equity value – are at a monthly frequency. Annual production and reserve data from year $t-1$ are used to predict monthly returns from July of year $t$ to June of year $t+1$ (the standard timing convention since Fama and French, 1992).

Specifically, we consider the following prediction equation,

$$\widehat{r_{t+1}^{S}-r_t} = \widehat{b}_0 + \widehat{b}_1 \ln(\widehat{q_{t+1}}) + \widehat{b}_2 \ln(q_t) + \widehat{b}_3(r_t) + \widehat{b}_4(\widehat{\tfrac{y_{t+1}}{x_{t+1}}}) + \widehat{b}_5(\tfrac{y_t}{x_t}) + \widehat{b}_6(\tfrac{q_t x_{t+1}}{p_t}) \qquad (16)$$

Only variables up to time $t$ (including the right-hand side variables in equation 13) are used to obtain the coefficients to forecast $\widehat{(y_{t+1}/x_{t+1})}$. The predicted value $\widehat{r_{t+1}^{S}-r_t}$ is derived from information at time $t$ or earlier. The approach may be employed in real time to forecast returns for the month ahead.

The predicted coefficient signs for equation (16) from the Proposition are $\widehat{b}_1 > 0, \widehat{b}_2 < 0$, $\widehat{b}_3 < 0$, $\widehat{b}_4 < 0$, $\widehat{b}_5 > 0$, and $\widehat{b}_6 > 0$. In addition, it follows from equation (12) that $\widehat{b}_1 = -\widehat{b}_2$. This reflects the Hotelling effect: it is the relative price increase that relates to investment returns.

We also have $\widehat{b_1} = 1$ as tested previously by Miller and Upton (1985), or in a more parsimonious regression that the coefficient on $\ln(\widehat{q_{t+1}}) - \ln(q_t) - r_t$ is equal to 1. Furthermore, $\widehat{b_5} \geq -\widehat{b_4}$ . This reflects the marginal cost of producing currently versus the next period. By keeping the resource in the ground one period longer, the future costs are reduced as they now are based on a (marginally) larger reserve. If this latter effect is considered negligible, we expect $\widehat{b_5} = -\widehat{b_4}$ .

We check first for multicollinearity in estimating equation (16). In particular, $\widehat{\frac{y_{t+1}}{x_{t+1}}}$ may be highly correlated with $\frac{y_t}{x_t}$, and $\ln(\widehat{q_{t+1}})$ with $\ln(q_t)$. Note first that estimation of $\widehat{\frac{y_{t+1}}{x_{t+1}}}$ employs time-varying firm-specific information and that $\ln(\widehat{q_{t+1}})$ employs time series information of 36 periods. As shown in Panel C of Table 4 the Variance Inflation Factors (VIFs) for each variable are less than 10 (the typical cutoff value for absence of multicollinearity) except for $\ln(\widehat{q_{t+1}})$ and $\ln(q_t)$ which have VIFs of 76.34 and 77.01, respectively. This implies that the standard errors of the coefficients for $\ln(\widehat{q_{t+1}})$ and $\ln(q_t)$ are, respectively, 8.74 and 8.78 times (square root of VIF) as large as without correlation between these variables themselves and the other regression variables. The coefficient estimates for these variables may be less reliable although this should not affect the overall forecast performance of the regression. To avoid this issue, as well as avoiding overfitting the forecast specification, we provide more parsimonious alternative specifications that combine variables under the assumption that the coefficient restrictions hold.

Panel A of Table 4 presents the stock return forecast results of a pooled OLS regression. Regression 1 *qualitatively* confirms the simple Hotelling Rule by which the expected commodity price increase implies a higher cost of equity capital (mean stock return). The expected future spot price and the current spot price have a significant, quantitatively similar but opposite, impact around 0.57 on stock returns of the commodity's producer. Regression 2 shows the parsimonious regression with the combined variable $\ln(\widehat{q_{t+1}}) - \ln(q_t) - r_t$. The coefficient is also 0.57, significantly positive (t-stat = 11.51). Panel C shows Chi-squared test results for the coefficient restrictions which cannot reject the restriction that $\widehat{b_1} = -\widehat{b_2}$. However, the restriction $\widehat{b_1} = 1$ is rejected statistically, and at 0.57 the coefficient is also economically less than 1. In the parsimonious specification of regression 2 the coefficient of 0.57 is also significantly smaller than 1.

Adding production and reserves, Regression 3 shows all coefficients as significant at the 1% level. They have the predicted signs, confirming all six predicted coefficient signs. The spot price effect is numerically again almost identical to the negative of the futures price impact and the chi-squared test in Panel C shows that the null hypothesis $\widehat{b_1} = -\widehat{b_2}$ cannot be rejected statistically. But again $\widehat{b_1} = 0.61$ is significantly less than 1. The predicted future production and current production coefficients are almost identical in absolute value. We have that $\widehat{b_5} > -\widehat{b_4}$ by a small amount, suggesting that the difference in current and future marginal costs, the resource exhaustion effect as represented by $c_x(y, x)$, is small. The chi-squared test in Panel C cannot reject the hypothesis that $\widehat{b_5} = -\widehat{b_4}$, confirming our prediction.

Regression 4 in Table 4 parsimoniously applies $\widehat{b_1} = -\widehat{b_2}$ and $\widehat{b_5} = -\widehat{b_4}$ (if $c_x(t)$ may be ignored).[20] We thus include only the excess expected future commodity price increase, the

[20] More precisely if $c_x(t) + r_t c_y(t)$ may be ignored, where *r* is the monthly risk-free rate so very small and $c_x(t)$ and $c_y(t)$ are of opposite sign, with $c_x(t)$ presumably of much smaller magnitude than $c_y(t)$.

production growth rate, and the reserves-to-equity value measure. The results are quantitatively like Regression 3, including that the excess commodity price increase coefficient equals 0.61 and is significantly less than 1.[21]

Focusing on Regression 4 we present the economic importance of the coefficients for predicting stock returns. The price coefficients of around 0.61 in magnitude imply that a one standard deviation increase in the expected commodity price increase (1.40 in Table 2) raises *monthly* stock returns by about 85 basis points. The production coefficient of around -9.2 implies that a one standard deviation boost in output as a fraction of reserves (0.076 in Table 2) lowers stock returns by about 70 basis points. Lastly, the reserves-to-equity value coefficient of 0.095 implies that a one standard deviation increase in the value of reserves as a fraction of market equity (5.33 in Table 2) would raise the future stock return by about 51 basis points. Accordingly, each of the three variables in the parsimonious regression is economically important in affecting stock returns.

The pure Hotelling effect suggests a 1% higher expected return if the expected commodity price increase is 1% higher. The result in regression 4, however, of 0.61% is significantly less than 1%. Although it has the predicted sign and is significantly different from zero, this is quantitatively lower than expected. A possible explanation is that the driving forces from the Hotelling model work but, in part, are moderated by elements omitted from the model,

[21] The R-squares for all pooled regressions in Table 4 are from 0.25% - 0.43% which appears low. However, they are for forecasts based on past variables, for monthly returns, and at the firm level. Predictability of stock portfolio returns at monthly horizons is very low (Fama and French 1988), no matter what variables are used. The reason is that expected returns vary little compared to realized returns at monthly frequencies. Individual firms exhibit even more random variability in returns than portfolios of firms. Zhang (2005, Table 5) finds that zero-investment portfolio returns of high-value stocks and shorting low-value stocks are explained by various portfolio averages of firm characteristics, with the R-squared ranging from 0.16% to 0.71%. Belo et al. (2020) generate much higher R-squares such as around 25% but this is in-sample for the market portfolio and a 5-year horizon. For a 1-year horizon out-of-sample, Belo et al.'s R-squared is only 0.42% for the market portfolio. In comparison, the R-squared of 0.41% for our main specification is quite high since it is out-of-sample for individual firms and a 1-month horizon. We also argue later in the paper that the economic significance of the 0.41% R-squared is sizeable.

or caused by measurement error in the forecast variables generating downward biased estimates. For instance, reduced expected commodity price growth may have a diminished impact on the stock return compared to what the model predicts if the firm may exercise its real option to shut down a mine. However, a more direct explanation may be that firms hedge their exposure to commodity price fluctuations. For instance, Acharya, Lochstoer, and Ramadorai (2013) find that, in the period from 2000 to 2010, 88% of fuel-producing firms hedged commodity price risk with derivatives. If firms, say, hedged 39% of their exposure to commodity price risk this would explain a reduction of the theoretical impact from one-to-one to the 0.61-to-one we find empirically.[22]

Panel B presents the results for the panel specification with firm-specific fixed effects. Here we also use the panel results for the predicted production-to-reserves ratio, $\widehat{\frac{y_{t+1}}{x_{t+1}}}$, based on Table 3. Overall, the results are much like the pooling case in Panel A. The main difference is for the parsimonious case where the coefficient on excess expected commodity price growth is smaller, equal to 0.34, again significantly below 1, but significantly positive at the 1% level. Further, the coefficients on the spot price and expected future spot price, while again quantitatively similar, are now statistically different from each other.

*Commodity differences*

Subdividing the mining industry into three main categories – precious metals, industrial metals, and energy fuel (oil and gas) – we further test if the industry category in which the mining firm is classified influences its expected return and predictability. Following the industry category distribution in our sample, we create two industry dummy variables –

[22] Our model is consistent with hedging but has little to say regarding this issue: The degree of hedging commodity price risk by the firm in the model based on maximizing stockholder wealth is indeterminate because investors may always hedge this risk on their own account equally effectively.

PreciousMetals and IndustrialMetals. We assign the PreciousMetals variable (set to one) to any observation with combined production value weight of gold and silver of more than 0.5, and similarly assign the IndustrialMetals dummy variable (set to one) to any firm with combined production value weight above 0.5 in zinc, copper, and nickel to represent the industrial metals category. The dummy coefficients may reflect differences in $Var(y_{t+1} / x_{t+1})$ across the industry categories, which we do not directly capture in our regressions.[23]

Because the production extraction processes and the nature of the commodities market may vary substantially between the three categories, their investment returns may have different relationships to the explanatory variables. To examine this possibility, we expand the category dummy variables to include interactions with the explanatory variables. We limit the specification to only the interactions with the variables in the parsimonious formulation in Table 4 (regression 4).

Table 5 (pooled) presents the results with the interaction dummies added to the parsimonious formulation from Panel A in Table 4. The interaction dummies capture the effects of structural differences in the cost functions related to variation in extraction technologies among the industry categories. The specification is equivalent to running separate regressions of the parsimonious formulation for the three commodity categories.

The excess expected commodity price growth coefficient increases to 0.78 and the expected production difference parameter more than doubles to -18.3 while the reserves-to-equity parameter is roughly unchanged when we add only the interaction dummy variables. When we also add the level dummy variables, the excess expected commodity price growth

[23] The empirical result with these industry dummies added is a case in-between the pooled and panel results of Table 4, Panels A and B, and presented in *Web Appendix W2*. The dummy coefficient is positive significant, at the 1% level for precious metals companies, which means they have higher expected returns than the oil and gas companies, all else equal. Other coefficients are similar to the pooled results in Table 4, Panel A.

coefficient becomes 0.85, the expected production difference parameter is -17.1, and the reserves-to-equity parameter raises to 0.085. For both cases (regressions 2 and 3 in Table 5), given the six interaction dummy coefficients, the three for precious metals are significant and the three for industrial metals are not.[24]

Precious metals have a significant and quantitatively large decreased sensitivity to the excess expected commodity price growth, reversing the overall sign. Thus, precious metals' cost of equity capital is negatively related to excess expected commodity price growth which is counter to the simple Hotelling rule and inconsistent with our model. Possibly our simple moving average predictor is inadequate for gold and silver prices. These prices may be hard to predict. One sign is that the gold and silver futures premium is not a good predictor of spot prices for gold and silver either.[25] The second interaction variable for precious metals is for the expected production growth difference, which is significantly positive and quantitatively large, and again reverses the negative sign predicted by our model. Conceivably for precious metals there are large inventories of the commodity (essentially any previously mined quantities of gold or silver) that compete with production. The final interaction variable for precious metals is negative. It indicates that for precious metals the impact of the reserves-to-equity ratio on stock returns is diminished, but in total still has the predicted negative sign. For industrial metals, the results are similar as for energy fuels, consistent with the model.

Table 5 (fixed effects) allows for firm-specific fixed effects, and accordingly omits the level dummy variables by industry category but retains the interaction dummies. The resulting

[24] The coefficient restriction tests and multicollinearity checks are consistent with the earlier results for the base case in Table 4 and are not tabulated. Specifically, even the expected commodity price growth coefficient of 0.85 is still significantly below 1. Apart from significant multicollinearity between $\ln(\widehat{q_{t+1}})$ and $\ln(q_t)$ there is no indication of multicollinearity for any of the variables, including the interaction variables.

[25] Chinn and Coibion (2014) find that futures prices for precious metals are poor predictors, whereas futures prices for fuel energy commodities are much better predictors of subsequent prices.

specification produces a qualitatively similar outcome to the pooled specification; however, the excess expected commodity price growth coefficient is now a bit smaller at 0.61 which is consistent with the results found in the pooling cases without interaction variables.[26]

*Prediction errors*

To evaluate how well our approach predicts future returns we obtain for each firm in the sample at each month the error from the forecasted return of regression equation (16) and compare it to the subsequent realized return one month later. We use the numbers from a recursive version (using only past information) of the parsimonious regression equation (4) shown in Table 4. We calculate the mean-squared forecast error (MSFE) by squaring the errors, averaging over all firms in the sample at each month, then taking the square root.

As a point of reference with other forecast methods, we also predict returns of each firm for one month ahead by their historical means. As demonstrated by Welch and Goyal (2008) and Cenesizoglu and Timmermann (2012) historical means generally forecast future mean returns better than typical forecast variables and statistical models, at least at the aggregate level. With the same approach as above but replacing the forecast from the parsimonious version of our model (using only past data) by the historical mean calculated over the same period (also using only past data). Then calculate the MSFE for the historical means as forecast. The results are displayed in Figure 1. We show the 12-month moving average of the monthly MSFE at each point as well as cumulated over time for the forecasts based on our modified Hotelling valuation (blue solid line) against the benchmark MSFE for the forecasts based on the historical means (green dashed line). The prediction errors for our model forecast are substantially lower than for the historical mean forecast (except for the end of the sample).

[26] Additional results, dealing with geographic differences of the mining firms, and results focusing on returns on assets instead of equity returns are available in *Web Appendix W2*.

As an alternative benchmark we also compare the forecast errors for our modified Hotelling Valuation model to a version of the traditional Hotelling Valuation (based on Miller and Upton, 1985) in which we omit the production and reserves variables and only use the expected spot rate increase net of the interest rate to forecast the excess stock returns. In Figure 1 the MSFE for the traditional Hotelling Valuation (red dotted line) is worse than for our modified Hotelling Valuation but better than the historical mean-based forecasts.

*5.3. Portfolio sorting*

For a further indication of economic importance and to identify the influence of traditional risk factors we sort at each time all mining firms in our sample by predicted stock returns. [27] To forecast return for time $t+1$, we use the fitted value from the parsimonious version of equation (16), regression 4 in Panel A of Table 4, using the coefficients based on each prior observation up to time $t$ along with the predictor variables at $t$, to sort firms into quintiles. We pool the first 24 time series data points across all firms (4485 data points) to estimate initial coefficient values, then roll forward using an expanding window. Quintile 1 in each month includes the observations (firms) with the 20% *lowest* predicted returns, and Quintile 5 in each month contains those with the 20% *highest* predicted returns. Subsequent monthly returns for each quintile are recorded and averaged over time.

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[27] The results from equation (16) may call in doubt the model's ability to obtain economically meaningful returns from sorting into quintiles because the R-squared is only 0.0041 (0.41 percent). However, an argument of Cochrane (2001, p. 447) allows us to estimate the sort of trading strategy returns that we may expect, even with a low R-squared. Since R-squared is the ratio of the explained return variance and total return variance, $R^2 = \sigma^2_{PRED} / \sigma^2_{RET}$ we find $\sigma_{PRED} = \sigma_{RET}\sqrt{R^2} = 15.22 * 0.064 = 0.975$ (numbers from Tables 2 and 4). Assuming normality, the highest quintile of predicted returns starts at a predicted mean return of $\mu + \sigma_{PRED} x$, with $\mu$ the unconditional average return and $x$ given by the point at which the standard normal cumulative distribution is 0.8: $F(x)=0.8$ which implies $x=0.84$. The average predicted return for the top quintile may be determined by using $E(x|x>0.84)=f(0.84)/[1-F(0.84)]=0.28/0.20=1.40$, where $f(x)$ is the standard normal density. Then, for the predicted return distribution, holding the top and shorting the bottom quintile for one month generates $\mu+1.40(0.975)-[\mu-1.40(0.975)] = 2.73$ percent a month. The result is clearly economically significant despite the seemingly low R-squared but is an upper bound because the parameters used for prediction are likely measured with error.

For the full sample of 47,852 firm-month observations available for forecasting the realized average returns by quintile are in Panel A of Table 6. As expected, for a (pseudo) out-of-sample test of the model, these returns increase monotonically (with a minor inversion at Quintiles 3 and 4) from Quintile 1, with the lowest predicted return, to Quintile 5, with the highest predicted return. The difference between the Quintile 5 mean return of 1.08 percent a month and the Quintile 1 mean return of -0.17 percent is 1.24 percent which is economically large and highly significant, with a t-statistic of 3.17. The compounded annualized mean return difference amounts to an annual return of 16 percent.[28] The result confirms our theoretical prediction and supports the view that mining firms change their risk exposure significantly over time, creating predictable variation in costs of capital.

Theoretically, the return differences between quintiles should be systematic risk premia. The question is if these risk premia are compensation for known risk factors. To check we apply standard risk models to the return series to calculate the risk-adjusted returns. We consider the Fama-French three-factor model, the Fama-French five-factor model, and the Fama-French five-factor model plus the momentum risk factor of Carhart. We also consider the carry (or basis) factor of Yang (2013), Szymanowska et al. (2014), and Bakshi, Gao, and Rossi (2019) added to each of the models to account specifically for systematic commodity price risk.[29] We find that these risk factors generally explain only a small part of the returns. In fact, for the Fama-French three-factor model the alphas are higher than the raw returns (not shown). Table 6 presents the risk-adjusted returns based on the five-factor model (Fama and French, 2015) and for the five-factor model together with the carry factor. In Panel A the

[28] The 1.24 percent return is a reasonable fraction (0.45) of the 2.73 percent return expected (see previous footnote) assuming normal returns and no coefficient estimation error.

[29] We construct the carry factor with data from Bloomberg, using the procedure in Bakshi, Gao, and Rossi (2019), as the zero-investment return of holding out of 29 different commodity futures the five commodities that are most backwardated and shorting the five commodities that are most in contango.

difference in the alphas between Quintiles 5 and 1 is only marginally reduced to 1.09 percent monthly for the five-factor model (t-stat of 2.73) and 1.17 percent monthly for the five-factor plus carry risk model (t-stat of 2.95).

Panel B in Table 6 presents the results based on the panel approach with fixed effects for each firm in the sample. We use here the rolling version of regression 4 in Panel B of Table 4 to forecast the returns. The results are even clearer in this case: the returns increase perfectly monotonically from Quintile 1 with the lowest predicted return to Quintile 5 with the highest predicted return. The difference between the Quintile 5 mean return of 1.19 percent a month and the Quintile 1 mean return of -0.36 percent is 1.55 percent (t-statistic of 4.67) which implies an annualized return of more than 20 percent.

*Benchmarks and industry differences*

To provide a perspective for the sorting results we compare them to the return differences from alternative forecasts also using past data: (i) the forecast benchmark of the historical mean returns and (ii) a forecast based on "traditional Hotelling Valuation" using only the expected commodity price increase, both for the pooling and fixed effects, regressions 1 and 4 in Table 5. For (i) the difference between Quintile 5 and Quintile 1 is 0.34 percent a month, reduced to 0.19 percent when adjusted for risk. For the pooled case of (ii) the difference is 0.60 percent, reduced to 0.26-0.30 percent after risk adjustment. For the fixed effects case the difference is 0.48 percent, reduced to 0.15-0.20 percent after risk adjustment. Substantially smaller than for the full model. Details are in *Web Appendix W4*.

Examining differences by commodity, we consider forecasts based on regressions 3 and 5 in Table 5 and regression 4 of Table 4. Accounting for commodity differences increases the sorting returns. Individually, fuel energy and precious metals generate significant return differences among the quintiles, while the differences are less for industrial metals and not statistically significant. See *Web Appendix W4*.

## 6. Conclusion

Our model implies and confirms empirically that the expected growth rate of the natural resource price has a positive effect on the expected returns and cost of capital of commodity-producing firms. At the same time, firm attributes – the reserves-value-to-market-equity ratio and the production-to-reserves ratio change – have a positive and negative impact, respectively, on the expected return. The higher expected price of natural resources makes commodity-producing firms retain more reserves underground to profit from the higher expected price which makes these firms more sensitive to any systematic risk factors. Firms with lower production growth imply more future risk sensitivity: with given reserves of a commodity-producing firm, lower production growth today means higher future production which increases the sensitivity of a commodity-producing firm to future shocks.

The theoretical contribution of the paper is to use the investment-based asset pricing approach to derive results in the Hotelling framework for commodities firms that hold for any equilibrium discount rate. There is no need to specify the systematic risk factors and the discount rate may vary over time. Empirically, it is this theoretical refinement that allows us to test the Hotelling Valuation Principle (HVP) in a new way.

The HPV of Miller and Upton (1985) was initially confirmed but fared worse in subsequent empirical studies. Adelman (1993) finds that the HVP-predicted reserves are only half of measured actual reserves. Identifying the cause of this departure from the theory is difficult. It may be model limitations or measurement and accounting biases. By "differencing" the approach – considering stock returns instead of stock prices – we avoid biases related to the level of measured reserves. We furthermore provide additional implications not available from the level approach.

For North American firms producing precious metals, industrial metals, and fuel energy we find that average stock returns correspond to the HVP. A higher futures premium and higher reserves-to-equity ratios imply higher expected returns, and higher expected production growth rate lowers expected stock returns. The data confirm the predicted coefficient signs for all six of the variables identified by our modified Hotelling model. These restrictions also hold if we subdivide the firms by type of commodity, with the partial exception of precious metals for which we observe discrepancies compared to the model predictions. We assess the quantitative importance of the results, constructing portfolios by sorting the firms according to the predicted returns from our model. The sorting results show quite large average return differences consistent with the model predictions. The degree to which the results follow the predictions of the equilibrium Hotelling model suggests that the average return differences represent differences in equilibrium expected returns.

Theoretically, the source is time variation in the firms' choices of exposure to risk factors. The investment-based asset pricing approach did not need to specify the risk factors, but virtually no part of the average sorting return differences between fifth and first quintiles, amounting to 16 to 20 percent annually, can be explained by standard consumption-based risk factors. Inasmuch as all stocks are affected by the same systematic factors, it appears that one or more key risk factors are still missing in the consumption-based perspective.

The model quantitatively falls short in one respect. The impact of the expected spot growth rate on the excess stock returns is numerically smaller than predicted (about 61% of the predicted value). Although larger than the comparable number obtained from traditional tests of the HPV, around 50% (Adelman 1993), the number is significantly below the expected 100% in all our specifications. Possible reasons are the stylized nature of the model. Future work examining expected stock returns of commodity-producing firms may benefit from adding real options, conjoining the analyst forecasts for future commodity prices suggested by Cortazar et al. (2019), and considering the impact of hedging strategies.

Hedging commodity prices is common for commodity producers (e.g., Acharya, Lochstoer, and Ramadorai, 2013) and may easily, even in the context of our present model, explain the apparent incomplete reaction of stock returns to commodity price changes. If such hedging has become more common over time, it may even explain why the reaction was found to be complete in the earlier work of Miller and Upton (1985). Hedging may also explain the difference between the risk-premia in commodity prices and the risk-premia for the shares of commodity-producing firms. Our analysis points at the risk premium components related to production and reserve attributes of the producers, which are absent for holders of refined commodities, but additionally the stock returns in commodity producing firms contain the effect of hedging commodity price risk which is a component that is suitably excluded in measuring the risk premium for holding refined commodities.

Apart from confirming the HPV from a different perspective, our investment-based analysis provides vital additional insights. First, production factors together matter quantitatively more than the commodity price aspects of the traditional HPV alone. The implication for investment purposes is that holding the shares of commodity-producing firms is not a great substitute for investing in commodities. Second, we find that unfamiliar systematic risk factors, and time variation in firm exposure to these factors, are important determinants of the stock returns of commodity producers.

## Table 1. Sample Descriptive Statistics by Commodity

We use all mining firms in the *Compustat Industry Specific Annual* database with available production and reserve data for the following commodities: gold, silver, copper, nickel, zinc, crude oil (WTI) and natural gas. Panel A provides the total number of firms in the sample for each of the commodities separated by country of incorporation. "Other" refers to mining companies incorporated outside of Canada or the U.S. but listed on a U.S. or Canadian stock exchange. Panel B provides the number of mining companies included in the sample aggregated across the commodities by year.

**Panel A: Sample Size by Commodity**

| Country<br>Commodity | Canada | U.S. | Other | Total |
|---|---|---|---|---|
| Copper | 16 | 3 | 1 | 20 |
| Crude Oil | 184 | 185 | 33 | 402 |
| Gold | 74 | 7 | 13 | 94 |
| Natural Gas | 138 | 146 | 7 | 291 |
| Nickel | 3 | 0 | 0 | 3 |
| Silver | 13 | 3 | 0 | 16 |
| Zinc | 5 | 0 | 0 | 5 |

**Panel B: Sample Size by Year**

| Country<br>Year | Canada | U.S. | Other | Total |
|---|---|---|---|---|
| 2001 | 62 | 104 | 10 | 176 |
| 2002 | 79 | 118 | 15 | 212 |
| 2003 | 97 | 121 | 18 | 236 |
| 2004 | 105 | 121 | 24 | 250 |
| 2005 | 114 | 124 | 26 | 264 |
| 2006 | 118 | 120 | 22 | 260 |
| 2007 | 132 | 120 | 24 | 276 |
| 2008 | 141 | 134 | 22 | 297 |
| 2009 | 138 | 143 | 23 | 304 |
| 2010 | 142 | 140 | 27 | 309 |
| 2011 | 132 | 131 | 28 | 291 |
| 2012 | 136 | 133 | 32 | 301 |
| 2013 | 137 | 145 | 30 | 312 |
| 2014 | 147 | 155 | 26 | 328 |
| 2015 | 145 | 147 | 26 | 318 |
| 2016 | 134 | 136 | 28 | 298 |
| 2017 | 121 | 126 | 26 | 273 |
| 2018 | 123 | 122 | 28 | 273 |

### Table 2. Summary Statistics of Returns and Predictor Variables

The sample period is from July 2001 to December 2018. Panel A presents the summary statistics for the full sample, aggregated across the seven commodities. Stock return is the average return over the sample period in percent per month. Interest rate is the 3-month US T-Bill rate return in percent per month. Spot price growth is the average growth rate in percent per month of the commodity prices determined as the one-month-ahead log spot price minus the log spot price times 100. y/x is the annual production (extraction) of each firm per unit of proven and probable reserves averaged across all years and commodities. qx/p is the value of reserves divided by market equity averaged across all firms and years. b/(p+b) is the average across all firms and period of debt as a fraction of the value of the firm, as a measure of leverage. Panels B, C, and D provide the same statistics separated into precious metals (gold and silver), fuel energy (oil and gas), and industrial metals (copper, nickel, and zinc). std represents the standard deviation of the variable across the sample; 10%, 50%, and 90% indicate the cumulative distribution values across the sample.

**Panel A: Summary Statistics–All Commodities**

| | mean | std | 10% | 50% | 90% |
|---|---|---|---|---|---|
| stock return | 0.591 | 15.215 | -18.688 | 0.201 | 19.141 |
| interest rate | 0.106 | 0.128 | 0.003 | 0.074 | 0.319 |
| spot price growth | 0.871 | 1.403 | -1.012 | 0.723 | 2.747 |
| y/x | 0.127 | 0.076 | 0.042 | 0.108 | 0.243 |
| qx/p | 4.654 | 5.331 | 0.265 | 2.167 | 10.800 |
| b/(p+b) | 0.340 | 0.227 | 0.056 | 0.303 | 0.683 |

**Panel B: Summary Statistics–Precious Metals**

| | mean | std | 10% | 50% | 90% |
|---|---|---|---|---|---|
| stock return | 1.212 | 16.036 | -18.276 | -0.602 | 22.273 |
| spot price growth | 0.813 | 1.012 | -0.662 | 1.054 | 1.937 |
| y/x | 0.105 | 0.080 | 0.030 | 0.076 | 0.243 |
| qx/p | 5.628 | 6.072 | 0.613 | 3.291 | 14.686 |
| b/(p+b) | 0.268 | 0.220 | 0.047 | 0.195 | 0.617 |

**Panel C: Summary Statistics–Fuel Energy**

| | mean | std | 10% | 50% | 90% |
|---|---|---|---|---|---|
| stock return | 0.408 | 15.050 | -18.917 | -0.183 | 18.286 |
| spot price growth | 0.883 | 1.471 | -1.124 | 0.715 | 2.836 |
| y/x | 0.131 | 0.074 | 0.050 | 0.113 | 0.244 |
| qx/p | 3.660 | 4.868 | 0.218 | 1.972 | 9.222 |
| b/(p+b) | 0.352 | 0.226 | 0.056 | 0.322 | 0.696 |

**Panel D: Summary Statistics–Industrial Metals**

| | mean | std | 10% | 50% | 90% |
|---|---|---|---|---|---|
| stock return | 2.119 | 16.051 | -17.878 | 0.577 | 22.999 |
| spot price growth | 1.083 | 1.402 | -0.635 | 0.846 | 2.968 |
| y/x | 0.088 | 0.072 | 0.029 | 0.062 | 0.196 |
| qx/p | 9.832 | 8.509 | 0.507 | 7.033 | 22.297 |
| b/(p+b) | 0.331 | 0.226 | 0.088 | 0.260 | 0.658 |

## Table 3. Results of Production Prediction Regressions

The production variable is the ratio of the annual extraction quantity by the company per unit of proven and probable reserves, $\widehat{y_{t+1} / x_{t+1}}$. The forecast variables are: the weighted log of the expected future price, $\ln(\widehat{q_{t+1}})$, and the log spot price, $\ln(q_t)$, associated with the commodities produced by the firm (weighted by value); the real risk-free interest rate, $r_t$; the firm's weighted production of each commodity divided by the proven and probable reserves, $y_t / x_t$; and the firm's value of the proven and probable reserves divided by its market equity, *qx/p*. The regressions are: a pooled regression using annual observations with the predictive values lagged by one year (pooled) and a panel regression with firm fixed effects using annual observations with the predictive values lagged by one year (panel). T-stats are given in parentheses. "*" indicates significant at the 10% level, "**" significant at the 5% level, and "***" significant at the 1% level.

| | $\widehat{y_{t+1} / x_{t+1}}$ | |
|---|---|---|
| | Pooled | Panel |
| const | 0.016*** | |
| | (4.22) | |
| ln(exp spot price) | -0.010 | -0.005 |
| | (-0.80) | (-0.87) |
| ln(spot price) | 0.011 | -0.000 |
| | (0.98) | (-0.02) |
| interest rate | 0.116 | 0.057 |
| | (1.39) | (0.67) |
| $y_t/x_t$ | 0.914*** | 0.461*** |
| | (57.14) | (19.27) |
| qx/p | -0.0016*** | -0.0021*** |
| | (-7.38) | (-7.16) |
| r-squared | 0.459 | 0.632 |
| no. observations | 4528 | 4528 |

## Table 4. Stock Return Prediction Regression

OLS regression results across all firms and periods. The dependent variable is the monthly stock return which is predicted monthly from lagged variables. The prediction variables are the log of the weighted expected commodities price for the next month based on the weights of the commodities the firm produces, $\ln(\widehat{q_{t+1}})$; the log of the weighted spot price based on the weights of the commodities the firm produces, $\ln(q_t)$; the real risk-free interest rate, $r_t$ (3-month T-Bill minus inflation rate); the current production level relative to the level of the firm's reserves, $y_t / x_t$; the forecast of future production relative to the level of reserves based on previous-year variables, $\widehat{y_{t+1} / x_{t+1}}$; the last annual observation of firm reserves valued at current spot prices relative to the current market value of the firm's equity, $q_t\, x_{t+1} / p_t$. The excess expected spot price, Δspot, equals $\ln(\widehat{q_{t+1}}) - \ln(q_t) - r_t$. The predicted production difference, Δprod, equals $\widehat{y_{t+1} / x_{t+1}} - (1+r_t)(y_t / x_t)$. Panel A shows results for pooled regressions and Panel B for panel regressions with firm-level fixed effects. Panel C provides variation inflation factor (VIF) statistics to detect multicollinearity, and chi-squared statistics to test parameter restrictions. T-stats are in parentheses. Standard errors are Shanken (1992)-adjusted for measurement error in the estimated variables. "*" indicates significant at the 10% level, "**" significant at the 5% level, and "***" significant at the 1% level.

**Panel A: Pooled Regression**

| Dependent Variable: $r^S_{t+1} - r_t$ | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| constant | -0.082 | -0.099 | -0.268 | -0.411*** |
| | (-0.46) | (-1.20) | (-0.95) | (-3.54) |
| ln(exp spot price) | 0.569*** | | 0.608*** | |
| | (11.42) | | (9.94) | |
| ln(spot price) | -0.569*** | | -0.608*** | |
| | (-11.43) | | (-9.96) | |
| interest rate | -0.735*** | | -0.681*** | |
| | (-3.27) | | (-2.50) | |
| Δspot | | 0.572*** | | 0.611*** |
| | | (11.51) | | (10.65) |
| $\widehat{y_{t+1} / x_{t+1}}$ | | | -9.181*** | |
| | | | (-4.12) | |
| $y_t / x_t$ | | | 9.984*** | |
| | | | (4.22) | |
| Δprod | | | | -9.183*** |
| | | | | (-4.42) |
| qx/p | | | 0.098*** | 0.095*** |
| | | | (3.44) | (3.40) |
| r-squared | 0.0027 | 0.0025 | 0.0043 | 0.0041 |
| number of observations | 52337 | 52337 | 52337 | 52337 |

**Panel B: Firm Fixed Effects Panel Regression**

| Dependent Variable: $r^S_{t+1} - r_t$ | | | | |
|---|---|---|---|---|
| | 1 | 2 | 3 | 4 |
| ln(exp spot price) | 0.360*** | | 0.421*** | |
| | (6.09) | | (4.71) | |
| ln(spot price) | -0.375*** | | -0.438*** | |
| | (-6.32) | | (-4.89) | |
| interest rate | -1.009*** | | -0.931*** | |
| | (-4.45) | | (-2.74) | |
| Δspot | | 0.300*** | | 0.342*** |
| | | (5.14) | | (4.76) |
| $\widehat{y_{t+1} / x_{t+1}}$ | | | -17.67*** | |
| | | | (-5.81) | |
| $y_t/x_t$ | | | 16.97*** | |
| | | | (4.51) | |
| Δprod | | | | -11.58*** |
| | | | | (-4.76) |
| qx/p | | | 0.132*** | 0.150*** |
| | | | (3.44) | (4.38) |
| r-squared | 0.0242 | 0.0219 | 0.0265 | 0.0235 |
| number of observations | 52337 | 52337 | 52337 | 52337 |

**Panel C: Multicollinearity and Coefficient Restrictions**

| Variable | $\ln(\widehat{q_{t+1}})$ | $\ln(q_t)$ | $r_t$ | $\widehat{\frac{y_{t+1}}{x_{t+1}}}$ | $\frac{y_t}{x_t}$ | $\frac{q_t x_{t+1}}{p_t}$ | Δspot | Δprod | $\frac{q_t x_{t+1}}{p_t}$ |
|---|---|---|---|---|---|---|---|---|---|
| VIF | 76.34 | 77.01 | 1.13 | 4.35 | 4.27 | 1.10 | 1.01 | 1.01 | 1.00 |

| | | pooled | | | | panel | | | |
|---|---|---|---|---|---|---|---|---|---|
| Restriction | | 1 | 2 | 3 | 4 | 1 | 2 | 3 | 4 |
| $b_1 = 1$ | $\chi^2$-stat | 74.56 | 74.45 | 60.92 | 60.71 | 117.32 | 144.87 | 94.97 | 127.06 |
| | p-value | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) | (0.00) |
| $b_1 = -b_2$ | $\chi^2$-stat | 0.001 | | 1.80 | | 113.29 | | 145.96 | |
| | p-value | (0.98) | | (0.18) | | (0.00) | | (0.00) | |
| $b_4 = -b_5$ | $\chi^2$-stat | | | 0.65 | | | | 0.18 | |
| | p-value | | | (0.42) | | | | (0.68) | |

## Table 5. Return Prediction Regression with Commodity Interaction Effects

Results based on a pooled OLS regression across all firms and periods. The dependent variable is the monthly stock return which is predicted monthly from lagged variables. The prediction variables are the log of the weighted expected commodities price for the next month based on the weights of the commodities the firm produces, $\ln(\widehat{q_{t+1}})$; the log of the weighted spot price based on the weights of the commodities the firm produces, $\ln(q_t)$; the real risk-free interest rate, $r_t$; the current production level relative to the level of the firm's reserves, $y_t / x_t$; the forecast of future production relative to the level of reserves based on previous-year variables, $\widehat{y_{t+1} / x_{t+1}}$; the last annual observation of firm reserves valued at current spot prices relative to the current market value of the firm's equity, $q_t\, x_{t+1} / p_t$. The excess expected spot price, Δspot, equals $\ln(\widehat{q_{t+1}}) - \ln(q_t) - r_t$. The predicted production difference, Δprod, equals $\widehat{y_{t+1} / x_{t+1}} - (1 + r_t)(y_t / x_t)$. PreciousMetals and Industrial Metals are the intercept dummy variables. In addition, the interactions of the commodity dummies with the explanatory variables are included. T-stats are in parentheses. Standard errors are Shanken (1992)-adjusted for measurement error in the estimated variables. "*" indicates significant at the 10% level, "**" significant at the 5% level, and "***" significant at the 1% level.

| | Dependent Variable: $r^S_{t+1} - r_t$ | | | | |
|---|---|---|---|---|---|
| | Pooled | | | Fixed Effects | |
| constant | -0.411*** | -0.331** | -0.663*** | | |
| | (-3.54) | (-2.16) | (-4.19) | | |
| PreciousMetals | | | 2.602*** | | |
| | | | (4.73) | | |
| IndustrialMetals | | | 0.047 | | |
| | | | (0.06) | | |
| Δspot | 0.611*** | 0.781*** | 0.851*** | 0.342*** | 0.612*** |
| | (10.65) | (9.80) | (10.96) | (4.76) | (6.35) |
| Δprod | -9.183*** | -18.28*** | -17.08*** | -11.58*** | -18.83*** |
| | (-4.42) | (-5.96) | (-5.77) | (-4.76) | (-5.54) |
| qx/p | 0.095*** | 0.060** | 0.085*** | 0.150*** | 0.131*** |
| | (3.40) | (1.83) | (2.57) | (4.37) | (2.99) |
| (PreciousMetals)· (Δspot) | | -0.956*** | -1.830*** | | -1.761*** |
| | | (-4.01) | (-6.17) | | (-5.16) |
| (PreciousMetals) ·(Δprod) | | 43.17*** | 34.87*** | | 35.69*** |
| | | (5.22) | (4.27) | | (3.63) |
| (PreciousMetals) ·(qx/p) | | 0.088** | -0.073* | | -0.049 |
| | | (2.08) | (-1.41) | | (-0.63) |
| (IndustrialMetals)· (Δspot) | | -0.129 | 0.058 | | -0.261 |
| | | (-0.39) | (0.15) | | (-0.60) |
| (IndustrialMetals) ·(Δprod) | | -12.86 | -13.016 | | -19.27* |
| | | (-1.09) | (-1.11) | | (-1.41) |
| (IndustrialMetals) ·(qx/p) | | 0.002 | 0.010 | | -0.054 |
| | | (0.04) | (0.15) | | (-0.53) |
| r-squared | 0.0041 | 0.0067 | 0.0082 | 0.0235 | 0.0267 |
| number of observations | 52337 | 52337 | 52337 | 52337 | 52337 |

## Table 6. Portfolio Sorting Returns

The average returns are shown by quintiles. The quintiles are sorted from low to high by the predicted returns and for each quintile we show the subsequent (one month later) realized return averaged over the pseudo-out-of-sample time periods (July 2003 – December 2018). To forecast the return for time $t+1$, we use the fitted value from equation (16), with dummy variables for the various specifications, from the coefficients based on all prior observations up to time $t$ along with the predictor variables at $t$, to sort all firms into quintiles. We use the first 24 time series data points to estimate initial coefficients and use an expanding window for subsequent estimation. Quintile 1 in each month includes the observations (firms) with the 20% *lowest* predicted returns, and Quintile 5 in each month contains the observations with the 20% *highest* predicted returns. The subsequent monthly returns for each quintile are recorded and averaged and listed as ret 1 through ret 5 for quintiles 1 through 5, respectively. "tstat" refers to the t-statistic for the test of significance of the return compared to 0. "alpha1" refers to the risk-adjusted return based on the five-factor model of Fama and French (2015). "alpha2" refers to the risk-adjusted return using the five-factor model of Fama and French (2015) plus the carry factor specific for commodity price risk based on Bakshi, Guo, and Rossi (2019). Panels A and B present the results for all firms sorted based on the predictions from Tables 4A(4) and 4B(4), respectively.

**Panel A: Portfolio Sorting - Full Sample (Pooled)**

| | ret 1 | ret 2 | ret 3 | ret 4 | ret 5 | ret 5-1 |
|---|---|---|---|---|---|---|
| mean | -0.165 | 0.093 | 0.621 | 0.499 | 1.078 | 1.243 |
| tstat | -0.287 | 0.161 | 1.054 | 0.803 | 1.767 | 3.173 |
| alpha1 | -0.810 | -0.649 | -0.258 | -0.407 | 0.277 | 1.088 |
| tstat | -1.629 | -1.359 | -0.535 | -0.757 | 0.493 | 2.731 |
| alpha2 | -1.007 | -0.817 | -0.410 | -0.590 | 0.165 | 1.172 |
| tstat | -2.069 | -1.735 | -0.859 | -1.113 | 0.294 | 2.946 |

**Panel B: Portfolio Sorting - Full Sample (Fixed Effects)**

| | ret 1 | ret 2 | ret 3 | ret 4 | ret 5 | ret 5-1 |
|---|---|---|---|---|---|---|
| mean | -0.358 | 0.285 | 0.398 | 0.605 | 1.191 | 1.549 |
| tstat | -0.617 | 0.505 | 0.673 | 1.009 | 1.968 | 4.677 |
| alpha1 | -1.073 | -0.428 | -0.481 | -0.308 | 0.437 | 1.510 |
| tstat | -2.178 | -0.905 | -0.991 | -0.595 | 0.790 | 4.449 |
| alpha2 | -1.259 | -0.613 | -0.666 | -0.469 | 0.341 | 1.599 |
| tstat | -2.601 | -1.324 | -1.400 | -0.914 | 0.615 | 4.737 |

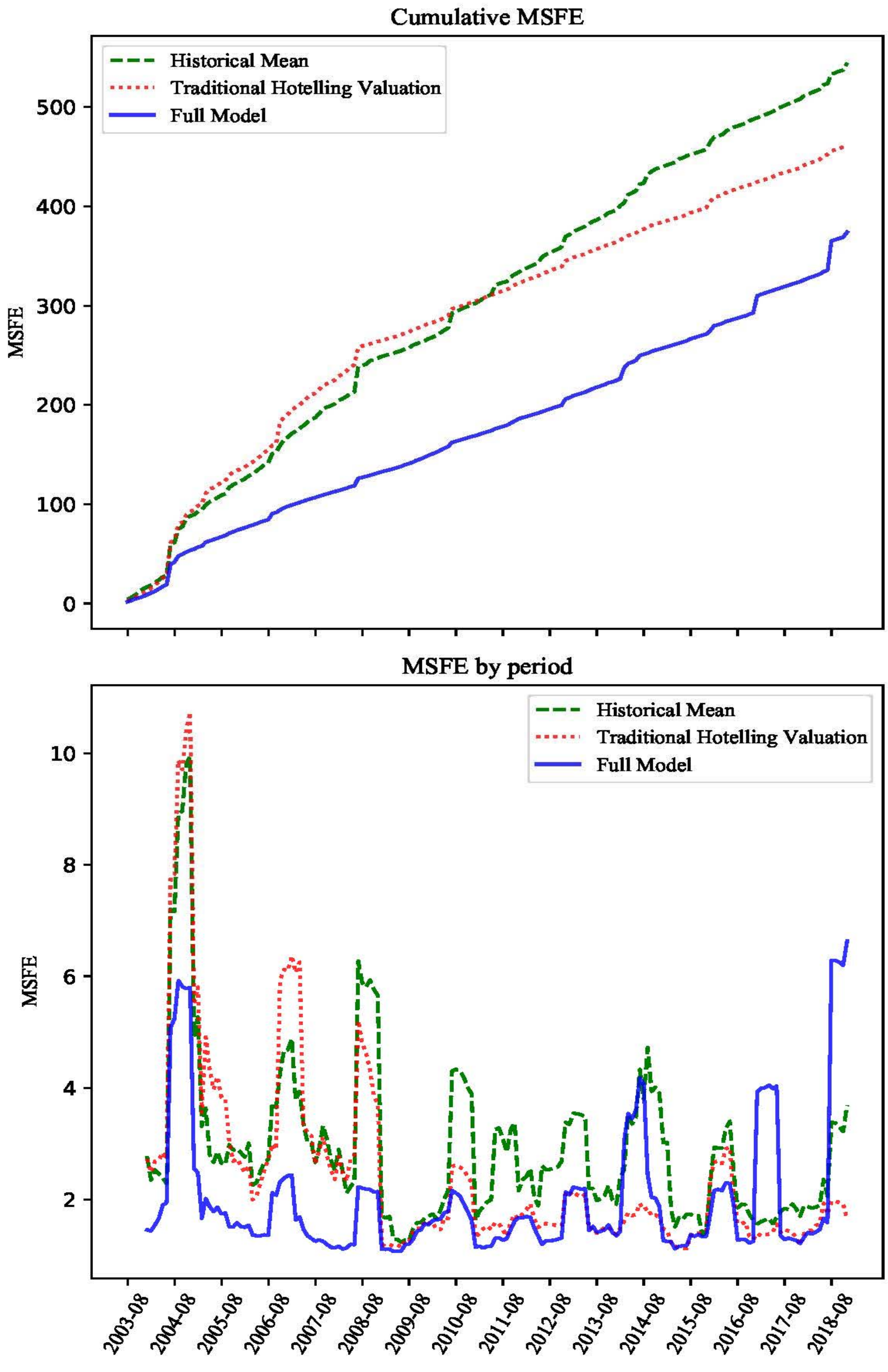
Cumulative MSFE
Historical Mean
Traditional Hotelling Valuation
Full Model
MSFE
0
100
200
300
400
500
MSFE by period
Historical Mean
Traditional Hotelling Valuation
Full Model
MSFE
2
4
6
8
10
2003-08
2004-08
2005-08
2006-08
2007-08
2008-08
2009-08
2010-08
2011-08
2012-08
2013-08
2014-08
2015-08
2016-08
2017-08
2018-08

Figure 1

## References

Acharya, V. V., Lochstoer, L.A., Ramadorai, T., 2013. Limits to Arbitrage and Hedging: Evidence from Commodity Markets. J. Financ. Econ. 109, 441–465. https://doi.org/10.1016/j.jfineco.2013.03.003

Adelman, M.A., 1990. Mineral Depletion, with Special Reference to Petroleum. Rev. Econ. Stat. 72, 1–10.

Adelman, M.A., 1993. Modelling World Oil Supply. Energy J. 14, 1–32.

Anderson, S.T., Kellogg, R., Salant, S.W., 2018. Hotelling under Pressure. J. Polit. Econ. 126, 984-1026. https://doi.org/10.1086/697203

Bakshi, G., Gao, X., Rossi, A.G., 2019. Understanding the Sources of Risk Underlying the Cross Section of Commodity Returns. Manage. Sci. 65, 619–641. https://doi.org/10.1287/mnsc.2017.2840

Balvers, R.J., Cosimano, T.F., McDonald, B., 1990. Predicting Stock Returns in an Efficient Market. J. Finance 45, 1109–1128.

Baur, D.G., 2014. Gold Mining Companies and the Price of Gold. Rev. Financ. Econ. 23, 174–181. https://doi.org/10.1016/j.rfe.2014.07.001

Beckmann, J., Belke, A., Czudaj, R., 2014. Does Global Liquidity Drive Commodity Prices? J. Bank. Financ. 48, 224–234. https://doi.org/10.1016/j.jbankfin.2014.04.007

Belo, F., Donangelo, A., Lin, X., Luo, D., 2020. What Drives Firms' Hiring Decisions? An Asset Pricing Perspective. Working Paper. https://doi.org/10.2139/ssrn.3569679

Berk, J.B., Green, R.C., Naik, V., 1999. Optimal Investment, Growth Options, and Security Returns. J. Finance 54, 1553–1607.

Bianchi, R.J., Drew, M.E., Fan, J.H., 2016. Commodities Momentum: A Behavioral Perspective. J. Bank. Financ. 72, 133–150. https://doi.org/10.1016/j.jbankfin.2016.08.002

Boons, M., Prado, M.P., 2019. Basis-Momentum. J. Finance 74, 239–279. https://doi.org/10.1111/jofi.12738

Cairns, R.D., Davis, G.A., 2001. Adelman's Rule and the Petroleum Firm. Energy J. 22, 31–54.

Cairns, R.D., Quyen, N.V., 1998. Optimal Exploration for and Exploitation of Heterogeneous Mineral Deposits. J. Environ. Econ. Manage. 35, 164–189. https://doi.org/10.1006/jeem.1998.1027

Cenesizoglu, T., Timmermann, A., 2012. Do Return Prediction Models Add Economic Value? J. Bank. Financ. 36, 2974–2987. https://doi.org/10.1016/j.jbankfin.2012.06.008

Chen, S.S., 2016. Commodity Prices and Related Equity Prices. Can. J. Econ. 49, 949–967. https://doi.org/10.1111/caje.12220

Chinn, M.D., Coibion, O., 2014. The Predictive Content of Commodity Futures. J. Futur. Mark. 34, 607–636. https://doi.org/10.1002/fut.21615

Cochrane, J.H., 1991. Production-Based Asset Pricing and the Link Between Stock Returns and Economic Fluctuations. J. Finance 46, 209–237.

Cochrane, J.H., 1996. A Cross-Sectional Test of an Investment-Based Asset Pricing Model. J. Polit. Econ. 104, 572–621.

Cochrane, J.H., 2001. Asset Pricing. Princeton University Press.

Cooper, I.A., Nyborg, K.G., 2006. The Value of Tax Shields IS Equal to the Present Value of Tax Shields. J. Financ. Econ. 81, 215–225. https://doi.org/10.1016/j.jfineco.2005.07.003

Cortazar, G., Millard, C., Ortega, H., Schwartz, E.S., 2019. Commodity Price Forecasts, Futures Prices, and Pricing Models. Manage. Sci. 65, 4141–4155.

Cox, J.C., Ingersoll, J.E., Ross, S.A., 1985. An Intertemporal General Equilibrium Model of Asset Prices. Econometrica 53, 363–384. https://doi.org/10.2307/1911241

Dar, A.B., Bhanja, N., Paul, M., 2019. Do Gold Mining Stocks Behave like Gold or Equities? Evidence from the UK and the US. Int. Rev. Econ. Financ. 59, 369–384. https://doi.org/10.1016/j.iref.2018.10.003

Daskalaki, C., Kostakis, A., Skiadopoulos, G., 2014. Are There Common Factors in Individual Commodity Futures Returns? J. Bank. Financ. 40, 346–363. https://doi.org/10.1016/j.jbankfin.2013.11.034

David, A., 2019. Exploration Activity, Long-Run Decisions, and the Risk Premium In Energy Futures. Rev. Financ. Stud. 32, 1536–1572. https://doi.org/10.1093/rfs/hhy067

Ellis, G.M., Halvorsen, R., 2002. Estimation of Market Power in a Nonrenewable Resource Industry. J. Polit. Econ. 110, 883–899. https://doi.org/10.1086/340777

Fama, E.F., French, K.R., 1988. Permanent and Temporary Components of Stock Prices. J. Polit. Econ. 96, 246–273.

Fama, E.F., French, K.R., 1992. The Cross-Section of Expected Stock Returns. J. Finance 47, 427–465. https://doi.org/10.1111/j.1540-6261.1992.tb04398.x

Fama, E.F., French, K.R., 2015. A Five-Factor Asset Pricing Model. J. Financ. Econ. 116, 1–22. https://doi.org/10.1016/J.JFINECO.2014.10.010

Gaudet, G., Khad, A.M., 1991. The Evolution of Natural Resource Prices under Stochastic Investment Opportunities : An Intertemporal Asset-Pricing Approach. Int. Econ. Rev. 32, 441–455.

Gorton, G.B., Hayashi, F., Rouwenhorst, K.G., 2013. The fundamentals of commodity futures returns. Rev. Financ. 17, 35–105. https://doi.org/10.1093/rof/rfs019

Hayashi, F., 1982. Tobin's Marginal q and Average q: A Neoclassical Interpretation. Econometrica 50, 223–224.

Hotelling, H., 1931. The Economics of Exhaustible Resources. J. Polit. Econ. 39, 137–175.

Huang, D., Kilic, M., 2019. Gold, Platinum, and Expected Stock Returns. J. Financ. Econ. 132(3), 50–75.

Hou, K., Xue, C., Zhang, L., 2015. Digesting anomalies: An Investment Approach. Rev. Financ. Stud. 28, 650–705. https://doi.org/10.1093/rfs/hhu068

Jacobs, H., Müller, S., 2020. Anomalies across the Globe: Once Public, no Longer Existent? J. Financ. Econ. 135, 213–230.

Jin, Y., Jorian, P., 2006. Firm Value and Hedging : Evidence from U.S. Oil and Gas Producers. J. Finance 61, 893–919.

Khan, S., Khokher, Z., Simin, T., 2017. The Information Content of a Nonlinear Macro-Finance Model for Commodity Prices. Rev. Financ. Stud. 30, 2818–2850. https://doi.org/10.1093/rfs/hhw087

Koijen, R.S.J., Moskowitz, T.J., Pedersen, L.H., Vrugt, E.B., 2018. Carry. J. Financ. Econ. 127, 197–225. https://doi.org/10.1016/j.jfineco.2017.11.002

Lin, C. C.-Y., Wagner, G., 2007. Steady-State Growth in a Hotelling Model of Resource Extraction. J. Environ. Econ. Manage. 54, 68–83. https://doi.org/10.1016/J.JEEM.2006.12.001

Lin, X., Zhang, L., 2013. The Investment Manifesto. J. Monet. Econ. 60, 351–366. https://doi.org/10.1016/J.JMONECO.2013.01.001

Linnainmaa, J.T., Roberts, M.R., 2018. The History of the Cross-Section of Stock Returns. Rev. Financ. Stud. 31, 2606–2649. https://doi.org/10.1093/rfs/hhy030

Litzenberger, R.H., Rabinowitz, N., 1995. Backwardation in Oil Futures Markets : Theory and Empirical Evidence. J. Finance 50, 1517–1545.

Liu, L.X., Whited, T.M., Zhang, L., 2009. Investment-based Expected Stock Returns. J. Polit. Econ. 117, 1105–1139. https://doi.org/10.1086/649760

Livernois, J., 2009. On the Empirical Significance of the Hotelling Rule. Rev. Environ. Econ. Policy 3, 22–41. https://doi.org/10.1093/reep/ren017

McDonald, S.L., 1994. The Hotelling Principle and In-Ground Values of Oil Reserves : Why the Principle Over-Predicts Actual Values. Energy J. 15, 1–17.

Miller, M.H., Upton, C.W., 1985. A Test of the Hotelling Valuation Principle. J. Polit. Econ. 93, 1–25.

Narayan, P.K., Narayan, S., Sharma, S.S., 2013. An Analysis of Commodity Markets: What Gain for Investors? J. Bank. Financ. 37, 3878–3889. https://doi.org/10.1016/J.JBANKFIN.2013.07.009

Pindyck, R.S., 1978. The Optimal Exploration and Production of Nonrenewable Resources. J. Polit. Econ. 86, 217–221.

Pindyck, R.S., Rotemberg, J.J., 1990. The Excess Co-Movement of Commodity Prices. Econ. J. 100, 1173–1189.

Ratti, R.A., Vespignani, J.L., 2015. Commodity Prices and BRIC and G3 Liquidity: A SFAVEC approach. J. Bank. Financ. 53, 18–33. https://doi.org/10.1016/J.JBANKFIN.2014.12.013

Restoy, F., Rockinger, G.M., 1994. On Stock Market Returns and Returns on Investment, J. Finance. 49, 543–556.

Schwerhoff, G., Stuermer, M., 2019. Non-Renewable Resources, Extraction Technology and Endogenous Growth. Working Paper. Federal Reserve Bank of Dallas. https://doi.org/10.24149/wp1506r1

Securities and Exchange Commision, 2021. Rules, Regulations and Schedules, Regulation S-K [17 CFR Part 229], https://www.sec.gov/divisions/corpfin/ecfrlinks.shtml (downloaded May 7, 2021).

Shanken, J., 1992. On the Estimation of Beta-Pricing Models. Rev. Financ. Stud. 5, 1–33. https://doi.org/10.1093/rfs/5.1.1

Shumlich, M., Wilsonl, C.A., 2009. Re-Examining The Hotelling Valuation Principle : Empirical Evidence from Canadian Oil and Gas Royalty Trusts. Working Paper, University of Saskatchewan.

Slade, M.E., 1982. Trends in Natural-Resource Commodity Prices: An Analysis of the Time Domain. J. Environ. Econ. Manage. 9, 122–137. https://doi.org/10.1016/0095-0696(82)90017-1

Slade, M.E., 1984. Tax Policy and the Supply of Exhaustible Resources: Theory and Practice. Land Econ. 60, 133–147.

Slade, M.E., Thille, H., 1997. Hotelling Confronts CAPM: A Test of the Theory of Exhaustible Resources. Can. J. Econ. 30, 685–708.

Slade, M.E., Thille, H., 2009. Whither Hotelling: Tests of the Theory of Exhaustible Resources. Annu. Rev. Resour. Econ. 1, 239–260. https://doi.org/10.1146/annurev.resource.050708.144223

Szymanowska, M., De Roon, F., Nijman, T., Van Den Goorbergh, R., 2014. An Anatomy of Commodity Futures Risk Premia. J. Finance 69, 453–482. https://doi.org/10.1111/jofi.12096

Tobin, J., 1969. A General Equilibrium Approach To Monetary Theory. J. Money Credit Bank. 1, 15–29.

Tufano, P., 1998. The Determinants of Stock Price Exposure : Financial Engineering and the Gold Mining Industry. J. Finance 53, 1015–1052.

Varian, H.R., 1992. Microeconomic Analysis, Third ed. Norton, New York, NY.

Welch, I., Goyal, A., 2008. A Comprehensive Look at The Empirical Performance of Equity Premium Prediction. Rev. Financ. Stud. 21, 1455–1508. https://doi.org/10.1093/rfs/hhm014

Yang, F., 2013. Investment Shocks and the Commodity Basis Spread. J. Financ. Econ. 110, 164–184. https://doi.org/10.1016/j.jfineco.2013.04.012

Zapata, H.O., Detre, J.D., Hanabuchi, T., 2012. Historical Performance of Commodity and Stock Markets. J. Agric. Appl. Econ. 44, 339–357. https://doi.org/10.1017/s1074070800000468

Zhang, L., 2005. The Value Premium. J. Finance 60, 67–103.

Zhang, L., 2017. The Investment CAPM. Eur. Financ. Manag. 23, 545–603. https://doi.org/10.1111/eufm.12129

Zhang, Y., 2015. The Securitization of Gold and its Potential Impact on Gold Stocks. J. Bank. Financ. 58, 309–326. https://doi.org/10.1016/j.jbankfin.2015.03.016

**Appendix**

The model in the text consists of the Bellman equation:

$$V(s_t) = \underset{y_t,\, b_{t+1}}{Max} \left\{ d_t + E_t[m_{t+1} V(s_{t+1})] \right\}, \tag{A1}$$

with constraints:

$$d_t = q_t\, y_t - c(y_t, x_t) - r_{t-1}\, b_t + (b_{t+1} - b_t), \quad x_{t+1} = x_t - y_t \;, \quad q_{t+1} = h(q_t, \varepsilon_{t+1}). \tag{A2}$$

The first-order condition for $y_t$ implies

$$q_t = c_y(y_t, x_t) + E_t[m_{t+1} V_x(x_{t+1}, q_{t+1}, b_{t+1})]. \tag{A3}$$

From the envelope theorem

$$V_x(s_t) = -c_x(y_t, x_t) + E_t[m_{t+1} V_x(s_{t+1})]. \tag{A4}$$

Combining (A3) and (A4) yields

$$V_x(s_t) = q_t - c_y(y_t, x_t) - c_x(y_t, x_t). \tag{A5}$$

Updating (A5) by one period and substituting into (A3) yields:

$$E_t\left[ m_{t+1} \left( \frac{q_{t+1} - c_y(y_{t+1}, x_{t+1}) - c_x(y_{t+1}, x_{t+1})}{q_t - c_y(y_t, x_t)} \right) \right] = 1. \tag{A6}$$

The first-order condition for the choice of debt $b_{t+1}$ generates:

$$E_t[m_{t+1}(1 + r_t)] = 1, \tag{A7}$$

which implies that $1 + r_t = 1 / E_t(m_{t+1})$. This is simply an equilibrium condition resulting from the fact that the debt is riskless. It follows that the model does not pin down the level of debt, $b_{t+1}$, and the firm's capital structure. We include debt in the model to allow us to conceptualize differences between returns on assets and stock returns and relate them empirically to the proper variables.

Given the equilibrium condition for stock returns, $E_t[m_{t+1}(1+r^S_{t+1})]=1$ and the definition of stock returns as $1+r^S_{t+1}=(p_{t+1}+d_{t+1})/p_t$ we have that

$$p_t = E_t[m_{t+1}(p_{t+1}+d_{t+1})]\,. \tag{A8}$$

This implies that $p_t = E_t[\sum_{j=1}^{\infty} m_{t+j}\, d_{t+j}]$, which means that $p_t$ is the ex-dividend market equity of the firm (normalizing the number of outstanding shares to one) so that $V(s_t)=d_t+p_t$ by comparison with equation (2). Given (A2),

$$p_t = E_t\{m_{t+1}[p_{t+1}+q_{t+1}y_{t+1}-c(y_{t+1},x_{t+1})-r_t\, b_{t+1}+(b_{t+2}-b_{t+1})]\}\,. \tag{A9}$$

Using the method of undetermined coefficients, guess that stock prices are proportional to the reserves and outstanding debt and then confirm that this guess is justified. For all *t*:

$$p_t = F_t\, x_{t+1} + G_t\, b_{t+1}\,. \tag{A10}$$

Substitute into (A9) and use (A3) as well as the property of the homogeneous cost function that $c(y_t,x_t)=c_y(y_t/x_t)\,y_t + c_x(y_t/x_t)\,x_t$. Then:

$$\begin{aligned} F_t\, x_{t+1} + G_t\, b_{t+1} = E_t\{m_{t+1}[&F_{t+1}\, x_{t+2} + G_{t+1}\, b_{t+2} + q_{t+1}(x_{t+1}-x_{t+2}) - r_t\, b_{t+1} + (b_{t+2}-b_{t+1}) \\ &- c_y(y_{t+1}/x_{t+1})(x_{t+1}-x_{t+2}) - c_x(y_{t+1}/x_{t+1})\, x_{t+1}]\} \end{aligned} \tag{A11}$$

Use equation (A6) $E_t\{m_{t+1}[q_{t+1}-c_y(y_{t+1}/x_{t+1})-c_x(y_{t+1}/x_{t+1})]\} = q_t - c_y(y_t/x_t)$ and equation (A7), $E_t[m_{t+1}(1+r_t)]=1$ to simplify the right-hand side of (A11):

$$\begin{aligned} &[F_t - q_t + c_y(y_t/x_t)]\, x_{t+1} + (G_t+1)\, b_{t+1} \\ &\quad = E_t\{m_{t+1}[F_{t+1} - q_{t+1} + c_y(y_{t+1}/x_{t+1})]\, x_{t+2} + (G_{t+1}+1)\, b_{t+2}\} \end{aligned} \tag{A12}$$

This equation is of the form $Z_t = E_t[m_{t+1}(Z_{t+1})]$ which has as the only non-bubble solution that $Z_t = 0$. Thus, we confirm the guessed solution and find that $F_t = q_t - c_y(y_t/x_t)$ and $G_t = -1$ for all *t*. Hence, we obtain equation (7) in the text

$$p_t = [q_t - c_y(y_t / x_t)]\, x_{t+1} - b_{t+1}\,. \tag{A13}$$

The stock return, $1 + r^S_{t+1} = (p_{t+1} + d_{t+1}) / p_t$, is obtained from (A2) and (A13), using $c(y_t, x_t) = c_y(y_t / x_t)\, y_t + c_x(y_t / x_t)\, x_t$:

$$1 + r^S_{t+1} = \frac{q_{t+1} - c_y(y_{t+1} / x_{t+1}) - c_x(y_{t+1} / x_{t+1}) - (1 + r_t)(b_{t+1} / x_{t+1})}{q_t - c_y(y_t / x_t) - (b_{t+1} / x_{t+1})}\ . \tag{A14}$$

Using (A13) and (A14) the excess return then equals, as given in equation (8) in the text:

$$r^S_{t+1} - r_t = \frac{\{[(q_{t+1} - q_t) / q_t] - r_t\} + [(1 + r_t)c_y(y_t / x_t) - c_y(y_{t+1} / x_{t+1}) - c_x(y_{t+1} / x_{t+1})] / q_t}{p_t / q_t x_{t+1}} \tag{A15}$$

Given the investment return from (A6): $1 + r^I_{t+1} = \dfrac{q_{t+1} - c_y(y_{t+1}, x_{t+1}) - c_x(y_{t+1}, x_{t+1})}{q_t - c_y(y_t, x_t)}$ (equal to the return on assets here), it is easy to confirm that

$$r^I_{t+1} = (1 - \lambda_t) r^S_{t+1} + \lambda_t r_t \ \ , \lambda_t = b_{t+1} / (p_t + b_{t+1})\,. \tag{A16}$$